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THESIS

**PERFORMANCE ANALYSIS OF DPSK SIGNALS WITH
SELECTION COMBINING AND CONVOLUTIONAL
CODING IN FADING CHANNEL**

by

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March, 1998

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COMBINING AND CONVOLUTIONAL CODING IN FADING CHANNEL**

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ABSTRACT

The performance analysis of a differential phase shift keyed (DPSK) communications system, operating in a Rayleigh fading environment, employing convolutional coding and diversity processing is presented. The receiver is the conventional square-law DPSK receiver using soft-decision convolutional decoding. The computationally efficient union bound technique is utilized to evaluate the system performance.

The coded and uncoded system performances of various diversity combining techniques are evaluated and compared. The combining techniques considered include equal gain combining (EGC), selection combining (SC), and a generalization of SC, whereby two or three signals with the two or three largest amplitudes are noncoherently combined. This generalized method is called second or third order SC and denoted as SC2 or SC3, respectively. Numerical results indicate that coded systems with SC2 and SC3 techniques significantly enhance the bit-error rate (BER) performance relative to that achievable with SC.

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I. INTRODUCTION

A. BACKGROUND

In the study of communication systems, the ideal additive white Gaussian noise (AWGN) channel is usually the starting point for the understanding of basic performance relationships. In this model, the primary source of performance degradation is the thermal noise generated by the receiver. In an ideal situation, we also infer that the signal attenuation versus distance behaves as if the propagation takes place over ideal free space.

However, for most practical radio channels, both the AWGN and the free space propagation model are inadequate to describe the communication channel and predict system performance. In a wireless communication system, a signal can travel from transmitter to receiver over multiple propagation paths. This effect can cause fluctuations in the received signal's amplitude, phase and angle of arrival, giving rise to multipath fading. A common model for describing the fading effects of many radio channels is the Rayleigh fading model.

It is well known that conventional digital transmission systems often provide poor performance over fading channels, with bursts of errors occurring during periods of low signal-to-noise ratio (SNR). While we could attempt to achieve good communication performance by increasing transmitter power or antenna size, these options are neither economically attractive nor, sometimes, possible. An alternative way to mitigate the effects of fading is the use of redundancy that can be provided by diversity.

Frequency diversity, time diversity and space diversity (by using multiple antennas) are the common methods for achieving diversity. Binary differential phase shift keying (DPSK) with diversity is considered in this thesis. There are a number of techniques available at the receiver to combine the signals from the diversity channels. For noncoherent combining, a commonly used technique is the equal gain combining (EGC), whereby all the diversity channels are noncoherently combined with equal gain. However, there is a drawback in the application of the EGC technique in multipath fading channels. With this technique, the receiver complexity is directly proportional to the number of resolvable channels L . From an implementation point of view, having the receiver complexity dependent on the characteristics of the physical channel is undesirable since L may vary with location as well as time. In addition, combining more signals using EGC does not necessarily enhance performance; a phenomenon called "noncoherent combining loss" can occur.

Another common diversity combining technique is selection combining (SC), which determines and selects the diversity channel with the highest SNR. Although SC is a suboptimal technique, the advantage of SC over EGC is the requirement of a less complex receiver. A thorough treatment of these two commonly used combining techniques can be found in [Ref. 1]. In recent years, new combining strategies, which provide significant improvements in system performance over that provided by SC, have been proposed [Ref. 2]. These methods include the second and third order selection combining techniques, denoted as SC2 and SC3, respectively. Instead of selecting only one channel with the maximum SNR, SC2 and SC3 techniques select the two and three channels with the first two and three maximum SNRs, respectively, for demodulation.

For further enhancement of the system performance in fading channel, forward error correction (FEC) techniques are normally employed in addition to diversity. Convolutional coding is one of the most widely used FEC techniques. The wide acceptance of this coding technique is mainly due to its simplicity in implementation and the relatively large coding gain that it can achieve. For the receiver, there are two types of convolutional decoding methods, namely hard-decision and soft-decision decoding. Between these two decoding techniques, soft-decision decoding provides more gain than the corresponding hard-decision decoding and is usually the preferred option.

The most useful techniques for estimating the performance of convolutional decoding are the union bounds and computer simulation. The usefulness of computer simulation is limited by the long computation times that are required to get a good statistical sample. The union bound is a technique that is computationally efficient. It provides a performance estimate accurate to within a small fraction of a decibel for all signal-to-noise ratios that give an error rate of 10^{-3} or less [Ref. 3].

B. OBJECTIVE

In this thesis, the performance of a coded binary DPSK receiver using coding, for different types of linear combining techniques, in Rayleigh fading environment are investigated. The use of the selection combining techniques requires a less complex receiver than the equal gain combining technique. The performance of the uncoded system has been previously examined in [Ref. 2]. The effects of applying soft-decision convolutional decoding in conjunction with these combining techniques are examined here.

Four different types of linear combining techniques are evaluated. These are: equal gain combining, selection combining, second order, and third order selection

combining techniques. First, the performances of the uncoded system using these combining techniques are reviewed in Chapter II. In [Ref. 2], the uncoded second order and third order selection combining techniques are found to offer significant improvements in system performance over that provided by the conventional selection combining technique. Chapter III examines the coded performances of the four combining techniques. Numerical results for these combining techniques are compared and discussed in Chapter IV.

II. PERFORMANCE OF AN UNCODED DPSK SYSTEM IN A RAYLEIGH FADING CHANNEL

In this chapter, we review the previous works on the performance of an uncoded DPSK receiver with various combining techniques in a Rayleigh fading channel.

At the receiver with L diversity channels, the SNR per bit, γ_b , is the sum of L instantaneous SNRs given by,

$$\gamma_b = \sum_{k=1}^L \gamma_k, \quad (1)$$

where γ_k is the instantaneous SNR of the k th diversity channel. Without loss of generality, we assume that the average SNR per diversity channel, $\bar{\gamma}_c$, is identical for all channels, that is, $\bar{\gamma}_c = E\{\gamma_k\}$. The average SNR per bit, $\bar{\gamma}_b$, is related to the average SNR per diversity channel, $\bar{\gamma}_c$, by the formula

$$\bar{\gamma}_b = L\bar{\gamma}_c. \quad (2)$$

The conditional bit error probability for a Rayleigh-faded binary DPSK system with b combined channels is given by [Ref. 4]

$$P_b(\gamma_b) = \frac{1}{2^{2b-1}} e^{-\gamma_b} \sum_{n=0}^{b-1} c_n \gamma_b^n, \quad (3)$$

where c_n is given by,

$$c_n = \frac{1}{n!} \sum_{k=0}^{b-1-n} \binom{2b-1}{k}. \quad (4)$$

The average bit error probability, P_b , can be determined by averaging the conditional bit error probability, $P_b(\gamma_b)$, over the probability density function (pdf), $f(\gamma_b)$, and is given by the formula,

$$P_b = \int_0^{\infty} P_b(\gamma_b) f(\gamma_b) d\gamma_b. \quad (5)$$

A. EQUAL GAIN COMBINING

In the binary DPSK receiver with EGC technique, all L diversity channels are selected for combining. The conditional bit error probability is given by (3) when b is set to L . For EGC, the pdf, $f(\gamma_b)$, is given by the formula [Ref. 4]

$$f(\gamma_b) = \frac{1}{(L-1)! \bar{\gamma}_c^L} \gamma_b^{L-1} e^{-\gamma_b/\bar{\gamma}_c}. \quad (6)$$

Substituting (3) and (6) into (5) and performing the integration, the average bit error probability is easily shown to be [Ref. 4]

$$P_b = \frac{1}{2^{2L-1} (L-1)! (1 + \bar{\gamma}_c)^L} \sum_{n=0}^{L-1} c_n (L-1+n)! \left(\frac{\bar{\gamma}_c}{1 + \bar{\gamma}_c} \right)^n. \quad (7)$$

In terms of average SNR per bit, $\bar{\gamma}_b$, the Equation (7) can be rewritten as

$$P_b = \frac{L^L}{2^{2L-1} (L-1)! (L + \bar{\gamma}_b)^L} \sum_{n=0}^{L-1} c_n (L-1+n)! \left(\frac{\bar{\gamma}_b}{L + \bar{\gamma}_b} \right)^n. \quad (8)$$

B. SELECTION COMBINING

In the case of selection combining, only one of the L diversity channels is selected. The coded conditional bit error probability becomes

$$P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b}, \quad (9)$$

which is the general equation for a binary DPSK receiver. In general, the pdf of γ_b is given by [Ref. 1]

$$f(\gamma_b) = L\alpha e^{-\alpha\gamma_b} (1 - e^{-\alpha\gamma_b})^{L-1} \quad (10)$$

where $\alpha = \frac{1}{\bar{\gamma}_c}$. Substituting (9) and (10) into (5) and performing the integration, the average bit error probability has been shown in [Ref. 2] to be

$$P_b = \frac{L}{2} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{\alpha}{1 + \alpha + \alpha k}. \quad (11)$$

In terms of the average SNR per bit, $\bar{\gamma}_b$, Equation (12) can be written as,

$$P_b = \frac{L^2}{2} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{1}{\bar{\gamma}_b + (1+k)L}. \quad (12)$$

C. SECOND ORDER SELECTION COMBINING

In the SC2 technique, we combine two channels. The conditional bit error probability is a special case of (3) with b set to 2,

$$P_b(\gamma_b) = \frac{1}{8} e^{-\gamma_b} \sum_{n=0}^1 c_n \gamma_b^n. \quad (13)$$

In general, the pdf of γ_b is given by [Ref. 2],

$$f(\gamma_b) = L(L-1)\alpha e^{-\alpha\gamma_b} \left[\frac{\alpha\gamma_b}{2} + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k \frac{1}{k} (1 - e^{-\alpha k \gamma_b / 2}) \right]. \quad (14)$$

Substituting (13) and (14) into (5) and performing the integration, the average bit error probability has been shown to be [Ref. 2]

$$P_b = \left(\frac{\alpha}{1+\alpha} \right)^2 \frac{L(L-1)}{8} \left\{ \frac{c_0}{2} + \frac{c_1}{1+\alpha} + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k B(k) \right\} \quad (15)$$

where

$$B(k) = \frac{(1+\alpha)[c_0(2+2\alpha+\alpha k) + 4c_1] + c_1\alpha k}{(2+2\alpha+\alpha k)^2}. \quad (16)$$

In terms of $\bar{\gamma}_b$, the average SNR per bit, Equations (15) and (16) can be written as

$$P_b = \left(\frac{L}{\bar{\gamma}_b + L} \right)^2 \frac{L(L-1)}{8} \left\{ \frac{c_0}{2} + \frac{c_1\bar{\gamma}_b}{\bar{\gamma}_b + L} + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k B(k) \right\} \quad (17)$$

and

$$B(k) = \frac{(\bar{\gamma}_b + L)[c_0(2\bar{\gamma}_b + (2+k)L) + 4c_1\bar{\gamma}_b] + c_1Lk\bar{\gamma}_b}{[2\bar{\gamma}_b + (2+k)L]^2} \quad (18)$$

respectively.

D. THIRD ORDER SELECTION COMBINING

The SC3 technique combine three channels. The conditional bit error probability is a special case of (3) with b set to 3

$$P_b(\gamma_b) = \frac{1}{32} e^{-\gamma_b} \sum_{n=0}^2 c_n \gamma_b^n. \quad (19)$$

The pdf of γ_b is given by [Ref. 2],

$$f(\gamma_b) = \frac{L(L-1)(L-2)}{2} \alpha e^{-\alpha \gamma_b} \left[\frac{\alpha^2 \gamma_b^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} (-1)^k \frac{1}{k^2} (k \alpha \gamma_b - 3(1 - e^{-\alpha k \gamma_b / 3})) \right]. \quad (20)$$

Substituting (19) and (20) into (5) and performing the integration, the average bit error probability is shown [Ref. 2] to be

$$P_b = \frac{L(L-1)(L-2)}{64} \alpha \sum_{m=0}^2 c_m \left[\frac{\alpha^2}{6} \frac{(m+2)!}{(1+\alpha)^{m+3}} + m! \sum_{k=1}^{L-3} \binom{L-3}{k} (-1)^k \frac{1}{k} V(k, m) \right] \quad (21)$$

where

$$V(k, m) = \frac{\alpha(m+1)}{(1+\alpha)^{m+2}} - \frac{3}{k(1+\alpha)^{m+1}} + \frac{3}{k \left(1 + \alpha + \frac{\alpha k}{3} \right)^{m+1}}. \quad (22)$$

In terms of average SNR per bit, $\bar{\gamma}_b$, Equations (21) and (22) can be written as

$$P_b = \frac{L^2(L-1)(L-2)}{64 \bar{\gamma}_b} \sum_{m=0}^2 c_m \left[\frac{L^2}{6} \frac{\bar{\gamma}_b^{m+1} (m+2)!}{(\bar{\gamma}_b + L)^{m+3}} + m! \sum_{k=1}^{L-3} \binom{L-3}{k} (-1)^k \frac{1}{k} V(k, m) \right] \quad (23)$$

and

$$V(k, m) = \left[\frac{L(m+1)}{(\bar{\gamma}_b + L)^{m+2}} - \frac{3}{k(\bar{\gamma}_b + L)^{m+1}} + \frac{3}{k[\bar{\gamma}_b + L(1 + k/3)]^{m+1}} \right] \bar{\gamma}_b^{m+1} . \quad (24)$$

Having reviewed the performance of an uncoded DPSK receiver operating in a Rayleigh fading channel, we will examine its coded performance in the next chapter.

III. PERFORMANCE OF A CODED DPSK SYSTEM IN A RAYLEIGH FADING CHANNEL

Coded bits are received by the receiver having a diversity of L . At the receiver, the SNR per code bit, Γ_b , is the sum of the L instantaneous SNRs given by,

$$\Gamma_b = \sum_{k=1}^L \Gamma_k, \quad (25)$$

where Γ_k is the instantaneous SNR of the k th diversity channel. Without loss of generality, we assume that the average coded SNR per diversity channel, $\bar{\Gamma}_c$, is identical for all channels, that is, $\bar{\Gamma}_c = E\{\Gamma_k\}$. The average SNR per coded bit, $\bar{\Gamma}_b$, is related to the average coded SNR per diversity channel, $\bar{\Gamma}_c$, by the formula

$$\bar{\Gamma}_b = L\bar{\Gamma}_c. \quad (26)$$

With the application of the FEC technique, the total SNR for k data bits is the same as the total SNR for n coded bits. The average SNR per coded bit, $\bar{\Gamma}_b$, is related to the average SNR per bit, $\bar{\gamma}_b$, by the formula,

$$\bar{\Gamma}_b = \frac{k}{n} \bar{\gamma}_b = r \bar{\gamma}_b \quad (27)$$

where $r = k/n$ is the code rate. Equating (26) and (27), the average SNR per diversity channel, $\bar{\Gamma}_c$, can be expressed in term of the average SNR per bit, $\bar{\gamma}_b$, as follows:

$$\bar{\Gamma}_c = \frac{r}{L} \bar{\gamma}_b. \quad (28)$$

The bit error probability for coded system employing convolutional codes is given by the union bound [Ref. 4]

$$P_b \leq \frac{1}{k} \sum_{d=d_{free}}^{\infty} w_d P_2(d) \quad (29)$$

where k is the number of information bit per level, w_d is the total information weight of all paths of weight d , d_{free} is the free distance and $P_2(d)$ is the probability of selecting a coded word that is distance d from all-zero code word. It has been shown in [Ref. 4] that $P_2(d)$ is equivalent to the bit error probability for a noncoherent binary system with a d th-order diversity.

A. EQUAL GAIN COMBINING

In a binary DPSK receiver with EGC technique, all the L diversity channels are selected for combining. The total effective diversity is Ld [Ref. 4] for the coded system and the conditional bit error probability, given the coded bit SNR, Γ_b , is a special case of (3) with b set to Ld ,

$$P_2(\Gamma_b, d) = \frac{1}{2^{2Ld-1}} e^{-\Gamma_b} \sum_{n=0}^{Ld-1} c_n \Gamma_b^n. \quad (30)$$

For EGC, the pdf of Γ_b in a Rayleigh fading channel is given by the formula [Ref. 4],

$$p(\Gamma_b) = \frac{1}{(Ld-1)! \bar{\Gamma}_c^{Ld}} \Gamma_b^{Ld-1} e^{-\Gamma_b/\bar{\Gamma}_c}. \quad (31)$$

We use the symbols $p(\Gamma_b)$ and $f(\gamma_b)$ to denote the pdfs of the coded and uncoded systems, respectively.

The bit error probability, $P_2(d)$, can be determined by averaging the conditional bit error probability $P_2(\Gamma_b, d)$ over the pdf $p(\Gamma_b)$ of Γ_b which is given by

$$P_2(d) = \int_0^{\infty} P_2(\Gamma_b, d) p(\Gamma_b) d\Gamma_b. \quad (32)$$

Substituting (30) and (31) into (32) and performing the integration, the bit error probability can be shown to equal [Ref. 2]

$$P_2(d) = \frac{1}{2^{2Ld-1} (Ld-1)! (1+\bar{\Gamma}_c)^{Ld}} \sum_{n=0}^{Ld-1} c_n (Ld-1+n)! \left(\frac{\bar{\Gamma}_c}{1+\bar{\Gamma}_c} \right)^n. \quad (33)$$

In terms of the average SNR per bit, Equation (33) can be written as,

$$P_2(d) = \frac{L^{Ld}}{2^{2Ld-1} (Ld-1)! (L+r\bar{\gamma}_b)^{Ld}} \sum_{n=0}^{Ld-1} c_n (Ld-1+n)! \left(\frac{r\bar{\gamma}_b}{L+r\bar{\gamma}_b} \right)^n. \quad (34)$$

The bit error probability of the coded system is obtained by substituting (34) into (29).

B. SELECTION COMBINING

In the case of selection combining, only one diversity channel is selected. The total effective diversity for the coded system is d and the conditional bit error probability, given the coded bit SNR Γ_b , is a special case of (3) with b set to d ,

$$P_2(\Gamma_b, d) = \frac{1}{2^{2d-1}} e^{-\Gamma_b} \sum_{n=0}^{d-1} c_n \Gamma_b^n. \quad (35)$$

The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b is the d -fold convolution of $f(\Gamma_b)$ given in (10) and repeated here for convenience

$$f(\Gamma_b) = L\alpha e^{-\alpha\Gamma_b} (1 - e^{-\alpha\Gamma_b})^{L-1} \quad (36)$$

where

$$\alpha = \frac{1}{\Gamma_c}. \quad (37)$$

Thus

$$p(\Gamma_b) = [f(\Gamma_b)]^{\otimes d} \quad (38)$$

where $\otimes d$ represents a d -fold convolution.

Let us denote the Laplace transform of $p(\Gamma_b)$ with $\Psi(s)$ given by

$$\Psi(s) = [\Phi(s)]^d \quad (39)$$

where $\Phi(s)$ is the Laplace transform of $f(\Gamma_b)$ given by (36). The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b can be obtained by the inverse Laplace transform of (39). There is no generic closed form solution [Ref. 5] for all possible values of d and L . An inverse transformation for each individual value of d and L can be easily carried out by the symbolic transformation package of MATHCAD [Ref. 6]. The probability $P_2(d)$ is obtained by substituting (35) and (38) into (32) and can be obtained by the numerical integration package of the software. The bit error probability of the coded system is obtained by substituting $P_2(d)$ into (29).

C. SECOND ORDER SELECTION COMBINING

In SC2, two diversity channels are selected for combining. The total effective diversity for the coded system is $2d$ and the conditional bit error probability, given the

coded bit SNR Γ_b , is obtained by setting b to be $2d$ in Equation (3),

$$P_2(\Gamma_b, d) = \frac{1}{2^{4d-1}} e^{-\Gamma_b} \sum_{n=0}^{2d-1} c_n \Gamma_b^n. \quad (40)$$

The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b is the d -fold convolution of $f(\Gamma_b)$ given in (14),

$$f(\Gamma_b) = L(L-1)\alpha e^{-\alpha\Gamma_b} \left[\frac{\alpha\Gamma_b}{2} + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k \frac{1}{k} (1 - e^{-\alpha k \Gamma_b / 2}) \right] \quad (41)$$

and

$$p(\Gamma_b) = [f(\Gamma_b)]^{\otimes d} \quad (42)$$

where $\otimes d$ represents a d -fold convolution. The Laplace transform of $p(\Gamma_b)$, denoted by $\Psi(s)$, is given by

$$\Psi(s) = [\Phi(s)]^d \quad (43)$$

where $\Phi(s)$ is the Laplace transform of $f(\Gamma_b)$ given in (41).

The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b can be obtained by the inverse Laplace transform of (43). There is no generic closed form solution for all possible values of d and L for the inverse transformation [Ref. 5]. The inverse transformation for individual values of d and L can be easily carried out using the symbolic transformation feature in MATHECAD. The probability, $P_2(d)$, is obtained by substituting (40) and (42) into (32) which is also determined by the numerical integration package of the software. The bit error probability of the coded system is obtained by substituting $P_2(d)$ into (29).

If we set $L = 2$, then the SC2 technique is equivalent to a EGC technique with a $2d$ -fold diversity. In this case, the pdf $f(\Gamma_b)$ given in (41) becomes

$$f(\Gamma_b) = \alpha^2 e^{-\alpha \Gamma_b} \Gamma_b. \quad (44)$$

The Laplace transform, $\Phi(s)$, of (44) is given by [Ref. 5],

$$\Phi(s) = \left(\frac{1}{1 + s\bar{\Gamma}_c} \right)^2. \quad (45)$$

Substituting (45) into (43), the Laplace transform $\Psi(s)$ of $p(\Gamma_b)$ becomes,

$$\Psi(s) = \left(\frac{1}{1 + s\bar{\Gamma}_c} \right)^{2d}. \quad (46)$$

By taking the inverse Laplace transform of (46), the pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b is found to be [Ref. 5],

$$p(\Gamma_b) = \frac{1}{(2d-1)! \bar{\Gamma}_c^{2d}} \Gamma_b^{2d-1} e^{-\Gamma_b/\bar{\Gamma}_c}. \quad (47)$$

Substituting (40) and (47) into (32), the probability, $P_2(d)$, becomes

$$\begin{aligned} P_2(d) &= \int_0^\infty \left[\frac{1}{2^{4d-1}} e^{-\Gamma_b} \sum_{n=0}^{2d-1} c_n \Gamma_b^n \right] \left[\frac{1}{(2d-1)! \bar{\Gamma}_c^{2d}} \Gamma_b^{2d-1} e^{-\Gamma_b/\bar{\Gamma}_c} \right] d\Gamma_b \\ &= \frac{1}{2^{4d-1} (2d-1)! \bar{\Gamma}_c^{2d}} \sum_{n=0}^{2d-1} c_n \int_0^\infty \Gamma_b^{n+2d-1} e^{-\Gamma_b(1+1/\bar{\Gamma}_c)} d\Gamma_b. \end{aligned} \quad (48)$$

The integral in (48) can be easily evaluated using the following identity [Ref. 7],

$$\int_0^{\infty} x^n e^{-\mu x} dx = n! \mu^{-n-1}. \quad (49)$$

Therefore,

$$\begin{aligned} P_2(d) &= \frac{1}{2^{4d-1}(2d-1)!\bar{\Gamma}_c^{2d}} \sum_{n=0}^{2d-1} c_n (2d-1+n)! \left(1 + \frac{1}{\bar{\Gamma}_c}\right)^{-n-2d} \\ &= \frac{1}{2^{4d-1}(2d-1)!(1+\bar{\Gamma}_c)^{2d}} \sum_{n=0}^{2d-1} c_n (2d-1+n)! \left(\frac{\bar{\Gamma}_c}{1+\bar{\Gamma}_c}\right)^n. \end{aligned} \quad (50)$$

As expected, Equation (50) is a special case of (33) with L set to 2. This verifies that the derivation on the probability $P_2(d)$ presented in this section is conceptually correct.

D. THIRD ORDER SELECTION COMBINING

In SC3, three diversity channels are selected for combining. The total effective diversity for the coded system is $3d$ and the conditional bit error probability, given the coded bit SNR Γ_b , is a special case of (3) with b set to $3d$,

$$P_2(\Gamma_b, d) = \frac{1}{2^{6d-1}} e^{-\Gamma_b} \sum_{n=0}^{3d-1} c_n \Gamma_b^n. \quad (51)$$

The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b is the d -fold convolution of $f(\Gamma_b)$ given by (20),

$$f(\Gamma_b) = \frac{L(L-1)(L-2)}{2} \alpha e^{-\alpha \Gamma_b} \left[\frac{\alpha^2 \Gamma_b^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} (-1)^k \frac{1}{k^2} (k\alpha \Gamma_b - 3(1 - e^{-\alpha k \Gamma_b/3})) \right] \quad (52)$$

and

$$p(\Gamma_b) = [f(\Gamma_b)]^{\otimes d} \quad (53)$$

where $\otimes d$ represents a d -fold convolution.

The Laplace transform of $p(\Gamma_b)$ denoted by $\Psi(s)$ is given by

$$\Psi(s) = [\Phi(s)]^d \quad (54)$$

where $\Phi(s)$ is the Laplace transform of $f(\Gamma_b)$ as given in (52).

The pdf $p(\Gamma_b)$ of the coded bit SNR Γ_b can be obtained by the inverse Laplace transform of (54). There is no generic closed form solution [Ref. 5] of the inverse transformation for all values of d and L . For each individual value of d and L , the inverse transformation can easily be carried out using the symbolic transformation feature of MATHCAD. The probability $P_2(d)$ is obtained by substituting (51) and (53) into (32), which can also be determined by numerical integration using Mathcad. The bit error probability of the coded system is obtained by substituting $P_2(d)$ into (29). If we set $L=3$, the SC3 technique is equivalent to a EGC technique with $3d$ -fold diversity. Following the same procedure described in the previous section, it can be verified that the probability $P_2(d)$ is a special case of (33) with L set to 3.

IV. NUMERICAL RESULTS AND DISCUSSION

The Laplace transforms of the pdf $f(\Gamma_b)$ of Γ_b in (36), (41) and (52) for the three selection combining techniques are listed in Appendix A. There are no general closed form expressions for the inverse Laplace transformations of the characteristic functions of $p(\Gamma_b)$, as given in (38), (43) and (54), for the three selection combining techniques. The inverse transformations for selected values of d and L have been carried out using the symbolic inverse Laplace transformation feature in MATHCAD. We evaluated the integral of (32) for all the three combining techniques using the numerical integration feature. Appendix B illustrates a procedure used to obtain reliable data.

Although the integrals in (32) have positive infinite limit, the functions within these integrals are nearly zero except in a small part of the domain. The major contribution to the integrals comes from a relatively small domain where the function is significantly different from zero. For the cases of interest, the range of such dominant domains spreads from 10 for $\bar{\gamma}_b = 0$ dB to 200 for $\bar{\gamma}_b = 20$ dB.

The expression for $p(\Gamma_b)$ contains terms which have coefficients of very large values, particularly for large d . For example (see Appendix B), we have determined $p(\Gamma_b)$ for the SC technique with $L=5$ and $d=10$. We note that the pdf, $p(\Gamma_b)$, contains 43 terms, each of which is the combination of an increasing power function and a decaying exponential function. We also note that most of the coefficients of $p(\Gamma_b)$ (see Appendix B) have very large values. The value of the exponential decaying function in the pdf is still relatively large for low values of Γ_b . Because of the limited precision of the computer, it is impossible to determine the exact differences (since $p(\Gamma_b)$ is less than one) between the large-value terms at low values of Γ_b . In fact, the expression of the pdf produces unstable results at low values of Γ_b , as shown in Fig. B1 of Appendix B. Thus, the lower limits for these integrals will have to be used carefully so as to minimize erroneous results.

On the other hand, if we select too high of an upper limit for the integrals, the functions within the integrals will appear as spikes to the numerical integration routine. Numerical integration packages in most application software, including MATHEMATICA and MATHCAD, will leave out the spikes returning a zero value as their contribution.

Taking these observations into consideration, we take care to determine both the lower and upper limits for the integrals. These two limits have to be determined for each value of $\bar{\gamma}_b$, d and L . A way to verify that the limits have been appropriately obtained is

to integrate them between their respective limits to ensure that the result is equal to or very near to 1 (see Appendix B).

A. PERFORMANCES OF UNCODED SYSTEMS

The average bit error probability for EGC and the three selection combining techniques are shown, for diversities up to $L = 5$, in Fig. 1 to Fig. 4. From these figures, the performances of the systems with smaller L are observed to be superior to those with larger L at lower values of $\bar{\gamma}_b$, denoted as E_b/N_o in the figures. This trend reverses as $\bar{\gamma}_b$ increases. This phenomenon can be attributed to the noncoherent combining loss. As mentioned before, for the noncoherent systems, these four combining techniques are not an optimal technique. At low value of $\bar{\gamma}_b$, the noise in each diversity channel will be significant and, from (2), more so for systems with larger L . As $\bar{\gamma}_b$ increases, the noise in each diversity channel becomes less and less significant. Consequently, each diversity channel signal contributes positively to the overall SNR, and therefore, systems with larger L will perform better than those with smaller L at high $\bar{\gamma}_b$ values. The effect of the noncoherent combining loss is more evident for systems with much larger diversity L , as shown in Fig. 5 to Fig. 7. From these figures, it is also noted that the performances of systems with larger L are only better than those with smaller L at much higher $\bar{\gamma}_b$ values.

B. PERFORMANCES OF CODED SYSTEMS

The performance of the coded system, for each one of the diversities, is compared with the uncoded system performance in Fig. 8 to Fig. 18. It is observed that there is a significant improvement in system performance with the application of convolutional coding. From these figures, it is also observed that the system performance improves with an increase in the constraint lengths, v . However, an increase in constraint length can only be accomplished by an increase in the complexity of the encoding and decoding circuitry.

The performance of the coded receiver, for each of the constraint lengths, is compared among the various diversities L in Fig. 19 to Fig. 23. Since the employment of the FEC technique is equivalent to adding more diversity to the systems, the effect of the noncoherent combining loss is more evident in these figures. This observation is similar to the one made for Fig. 5 to Fig. 7 for the uncoded systems with much higher diversity L . This implies that, using the FEC technique, it is more advantageous to use systems with lower diversity, such as $L = 2$ or 3, when operating in the low $\bar{\gamma}_b$ region.

The performance of the coded system for each constraint length and diversity is compared in Fig. 24 to Fig. 29. From these figures, it is easily observed that there is a significant performance improvement for SC2 and SC3 techniques relative to the SC technique. From Fig. 25 to Fig. 27 we can observe that the performance of SC2 provides as good of a performance as SC3 (or, equivalently, EGC for $L = 3$). In fact, the performance of the SC2 technique is slightly superior than that of the SC3 technique at low values of $\bar{\gamma}_b$. In Fig. 28 when $L = 4$, although there is a slightly wider difference between the coded performances of systems with SC2 and SC3 techniques, both techniques are found to provide significant performance improvement over that provided by the SC technique. It is also observed that their performances are also comparable to that of the EGC technique.

V. CONCLUSIONS

The performance of a coded system using linear combining techniques operating in Rayleigh fading has been presented. Previous results [Ref. 2] for the uncoded systems were verified. Convolutional encoding with soft-decision decoding provides an efficient means for obtaining performance improvement over Rayleigh fading channels. We note that the system performance of the square-law DPSK receiver is greatly enhanced by the application of the forward error correction technique. An increase in the constraint length produces a noticeable improvement in system performance.

A procedure to obtain reliable results using the numerical methods has been discussed. We have demonstrated that, if the effect of the round-off errors in numerical methods is not taken into consideration, the data obtained from the numerical integration can be erroneous. The upper limits of the integrals have also been appropriately determined to avoid the 'spike' effect. Only numerical results that satisfy the basic property of a pdf have been used for analysis.

We examined the performances of the uncoded systems using EGC, SC, SC2 and SC3. We note that the performance of the uncoded system is subject to a noncoherent combining loss, where a system with higher diversity will perform better than one with lower diversity only at high SNR. We have demonstrated that the noncoherent combining loss is more evident in systems with a much higher diversity.

We investigated the performance of a coded system for three selection combining techniques as a function of constraint lengths. The application of the FEC technique is equivalent to providing more diversity to the coded system. It is noted that the performance of system with a larger L is only superior to a system with smaller L at much higher SNRs. Hence, with the FEC technique it is desirable to use a system with low diversity if it is to be operated in a low SNR environment.

Finally, the performance of the coded system relative to the three selection combining techniques is compared as a function of the constraint length and diversity. Numerical results indicate that SC2 and SC3 provide significant improvement in system performance over that provided by SC. The performances of the coded systems with SC2 and SC3 techniques are also found to be comparable to that of the EGC.

In conclusion, we recommend that the SC2 and SC3 techniques with forward error correction methods should be used if possible. If the systems are to be operated in the low SNR region, low diversity should be employed for better performance. SC2 and SC3 techniques provide significant improvement in system performance over the SC

technique. While the system performances of the SC2 and SC3 techniques are comparable to that of the EGC technique, they require a much less complex receiver structure as compared to EGC.

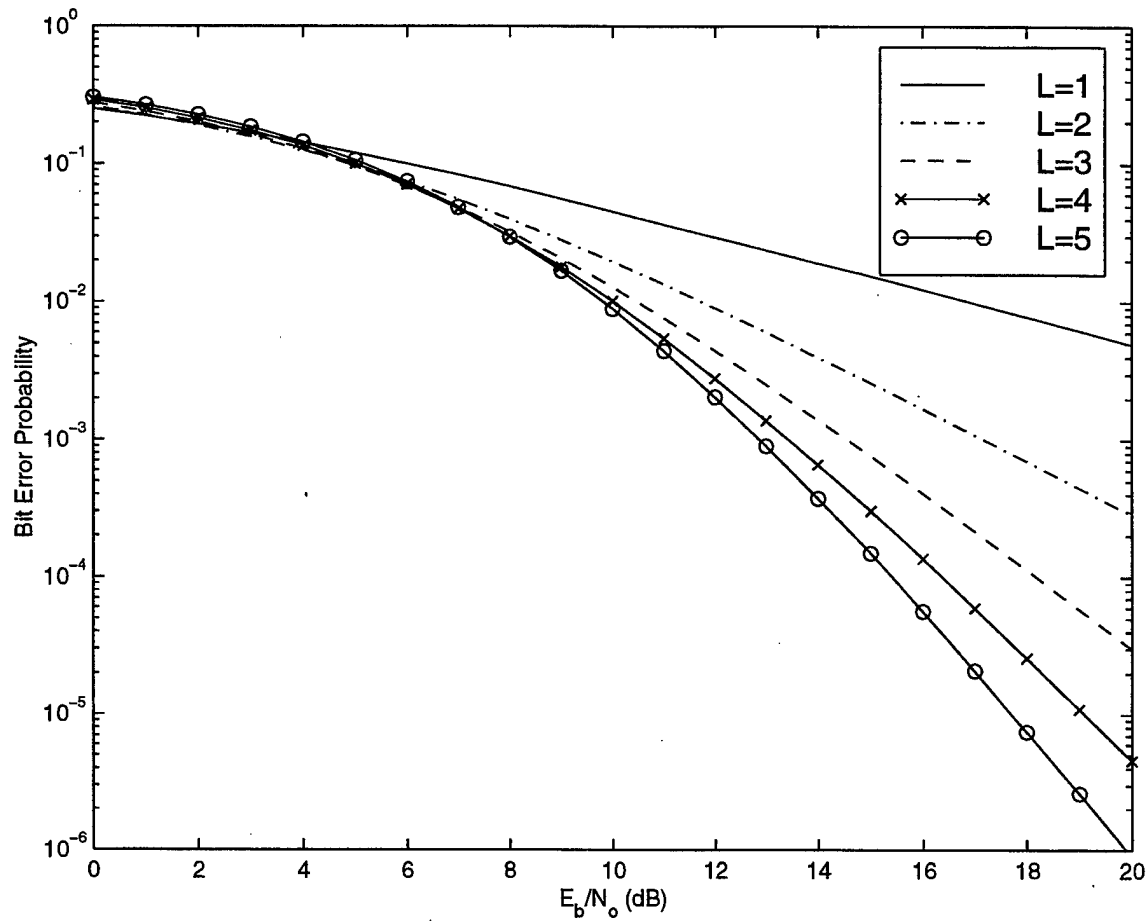


Figure 1. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Equal Gain Combining Technique for Diversities $L= 1, 2, 3, 4$ and 5 .

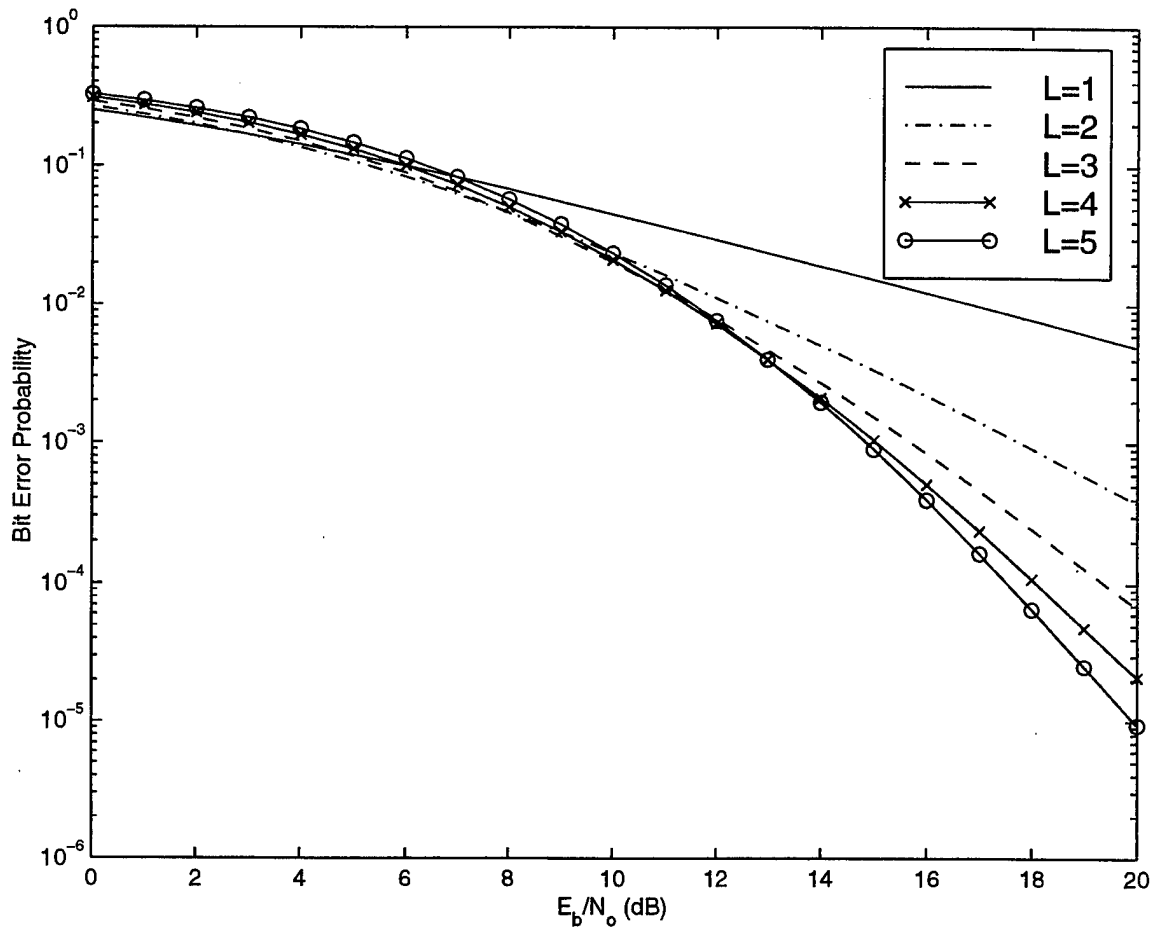


Figure 2. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Selection Combining Technique for Diversities $L=1, 2, 3, 4$ and 5 .

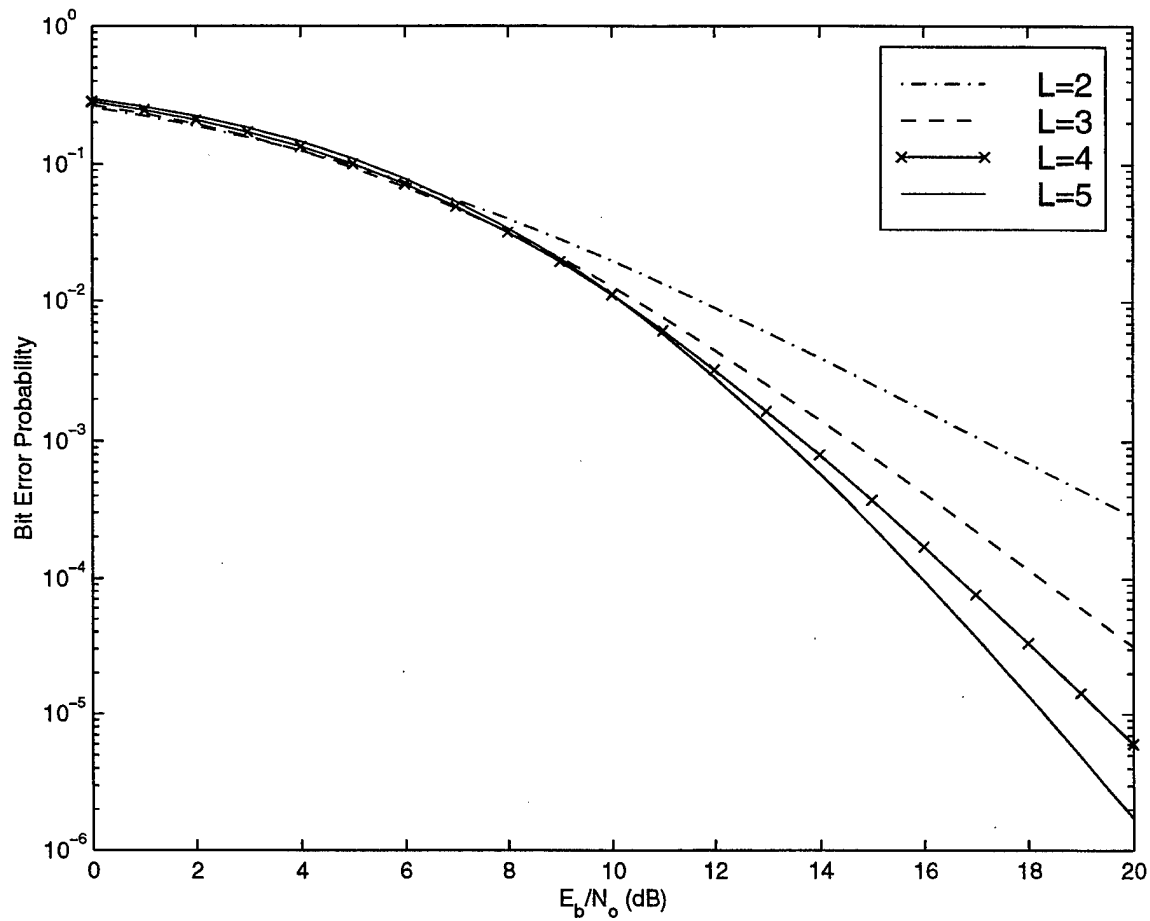


Figure 3. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique for Diversities $L = 2, 3, 4$ and 5 .

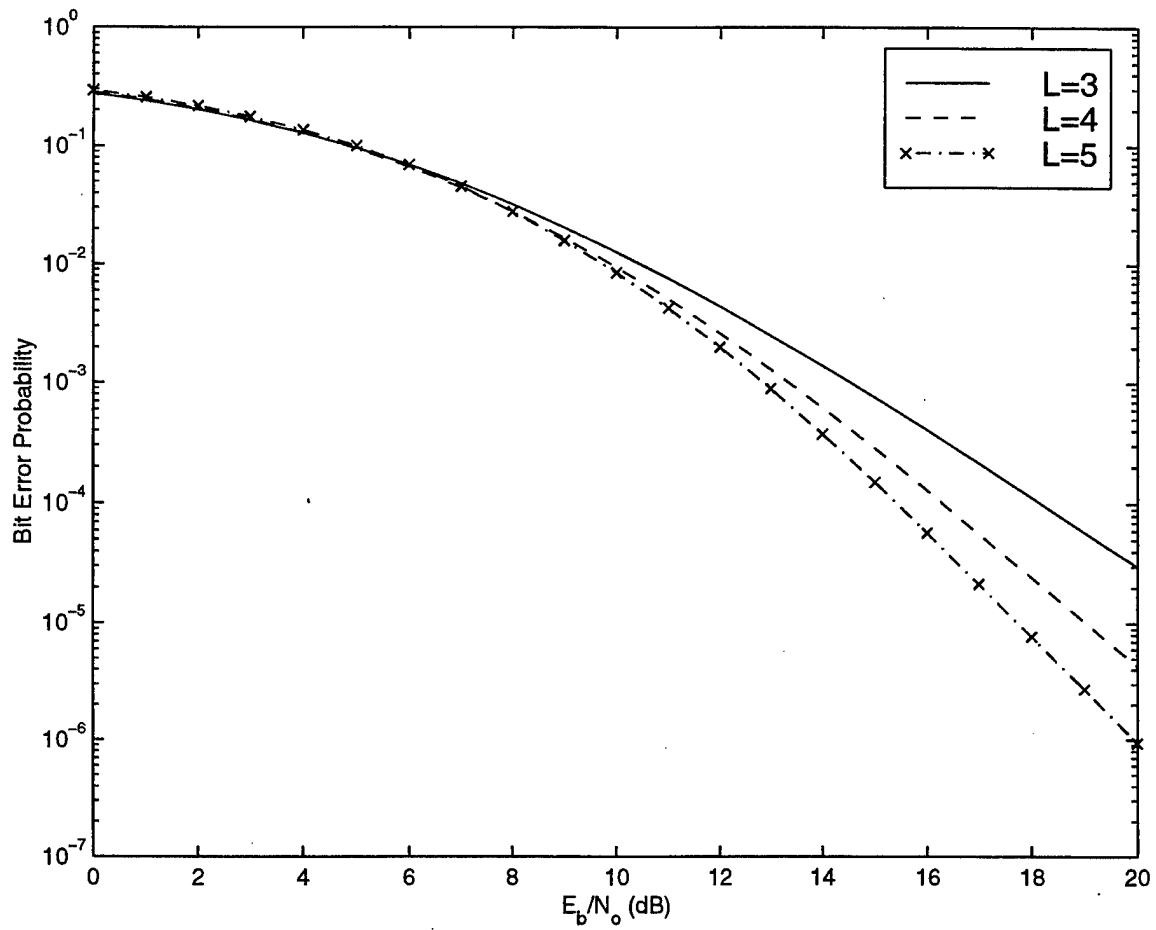


Figure 4. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Third Order Selection Combining Technique for Diversities $L = 3, 4$ and 5 .

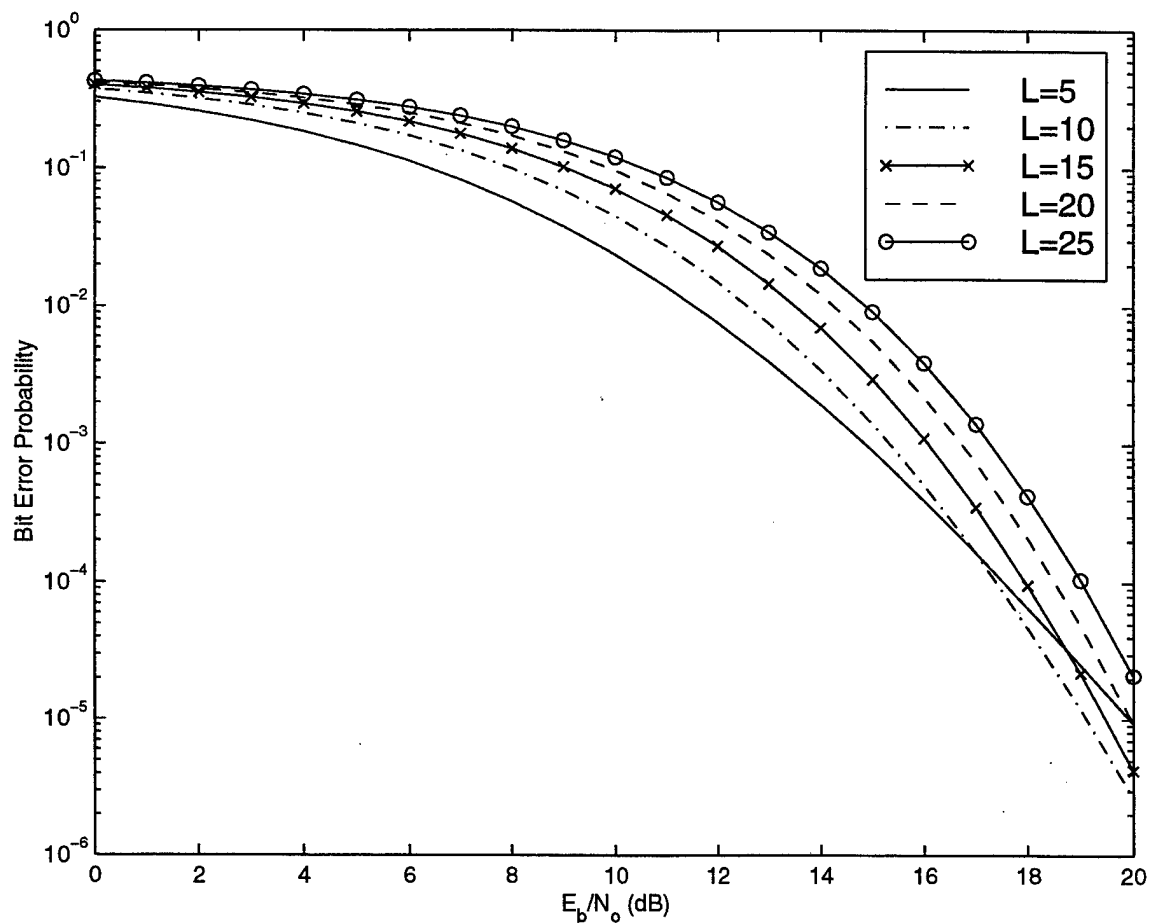


Figure 5. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Selection Combining Technique for Diversities $L = 5, 10, 15, 20$ and 25 .

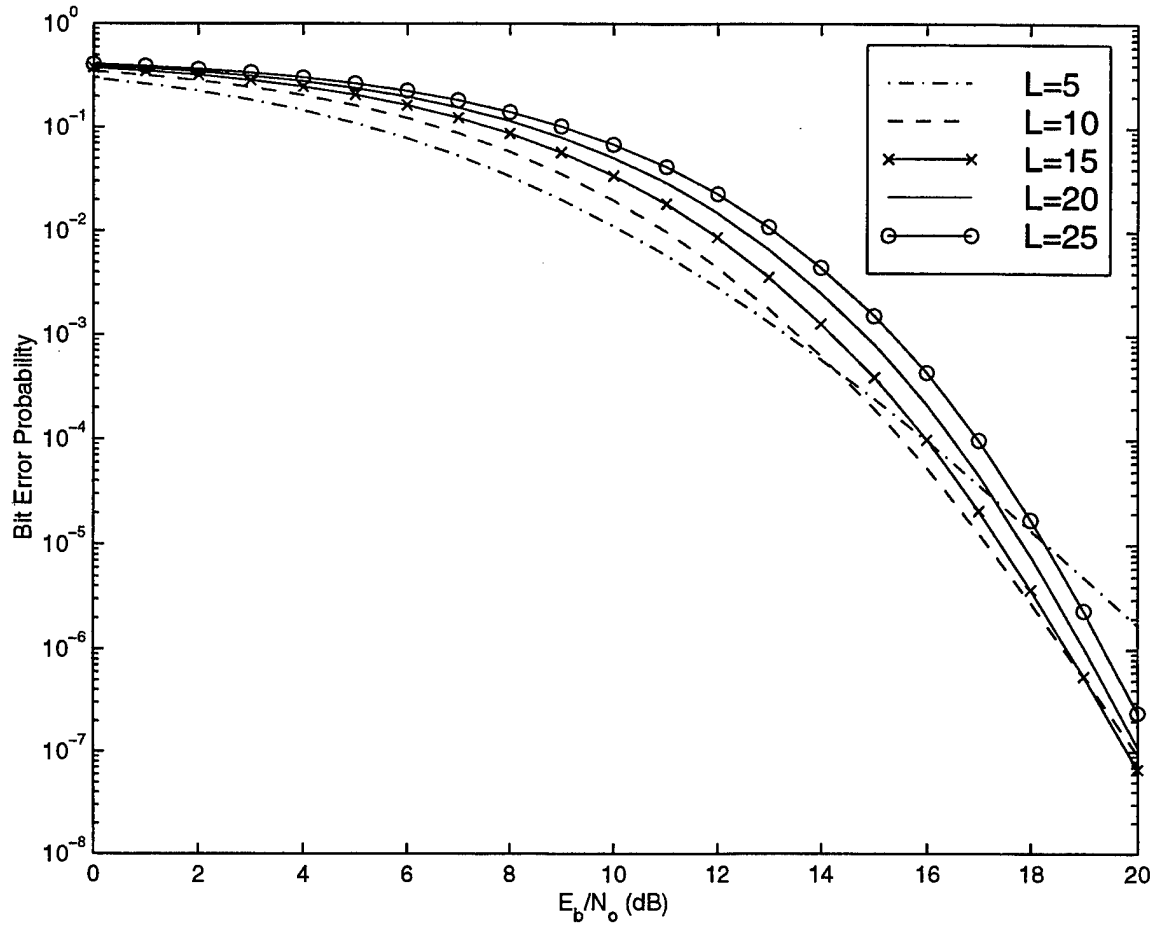


Figure 6. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique for Diversities $L = 5, 10, 15, 20$ and 25 .

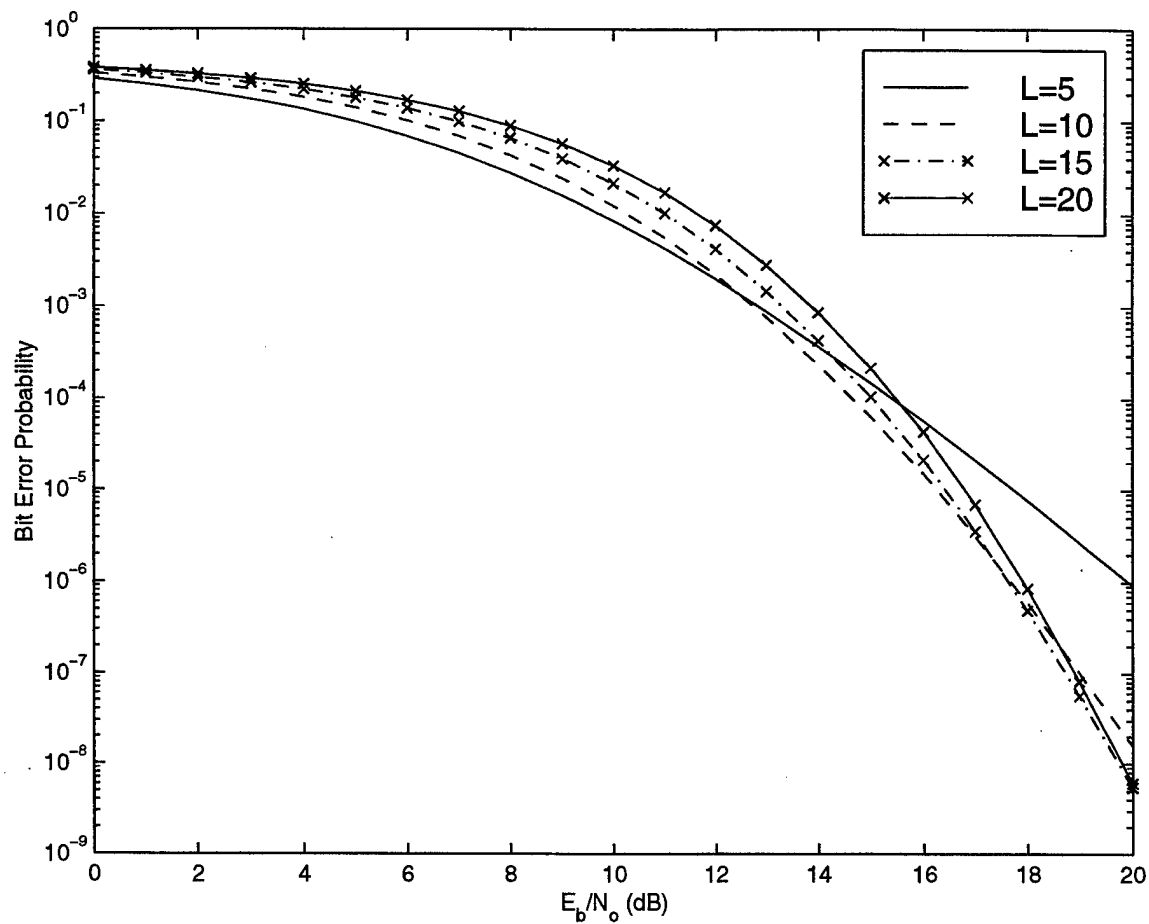


Figure 7. Performance of a Non-Coherent Uncoded DPSK Receiver in Rayleigh Fading With Third Order Selection Combining Technique for Diversities $L = 5, 10, 15$ and 20 .

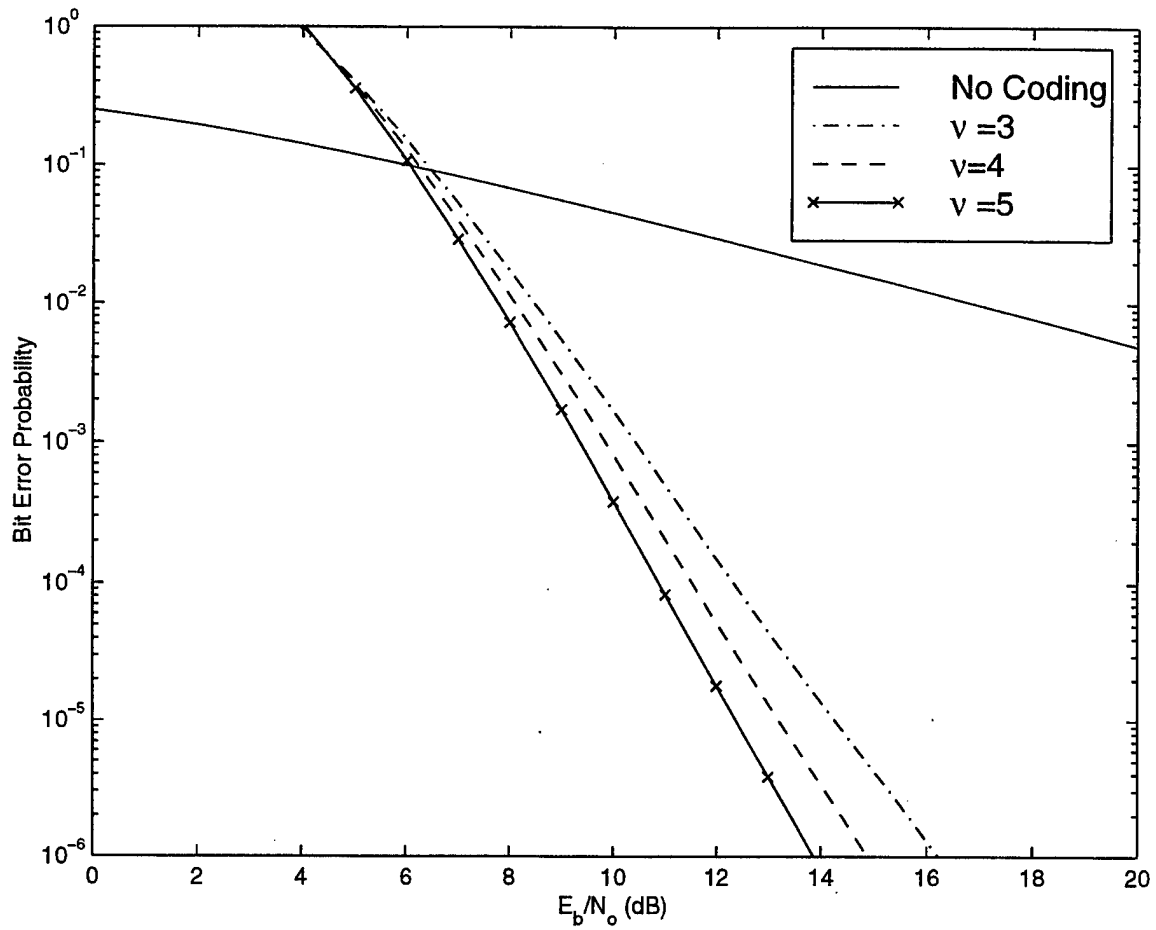


Figure 8. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate 1/2, Diversity $L = 1$ and Constraint Length $v = 3, 4$ and 5.

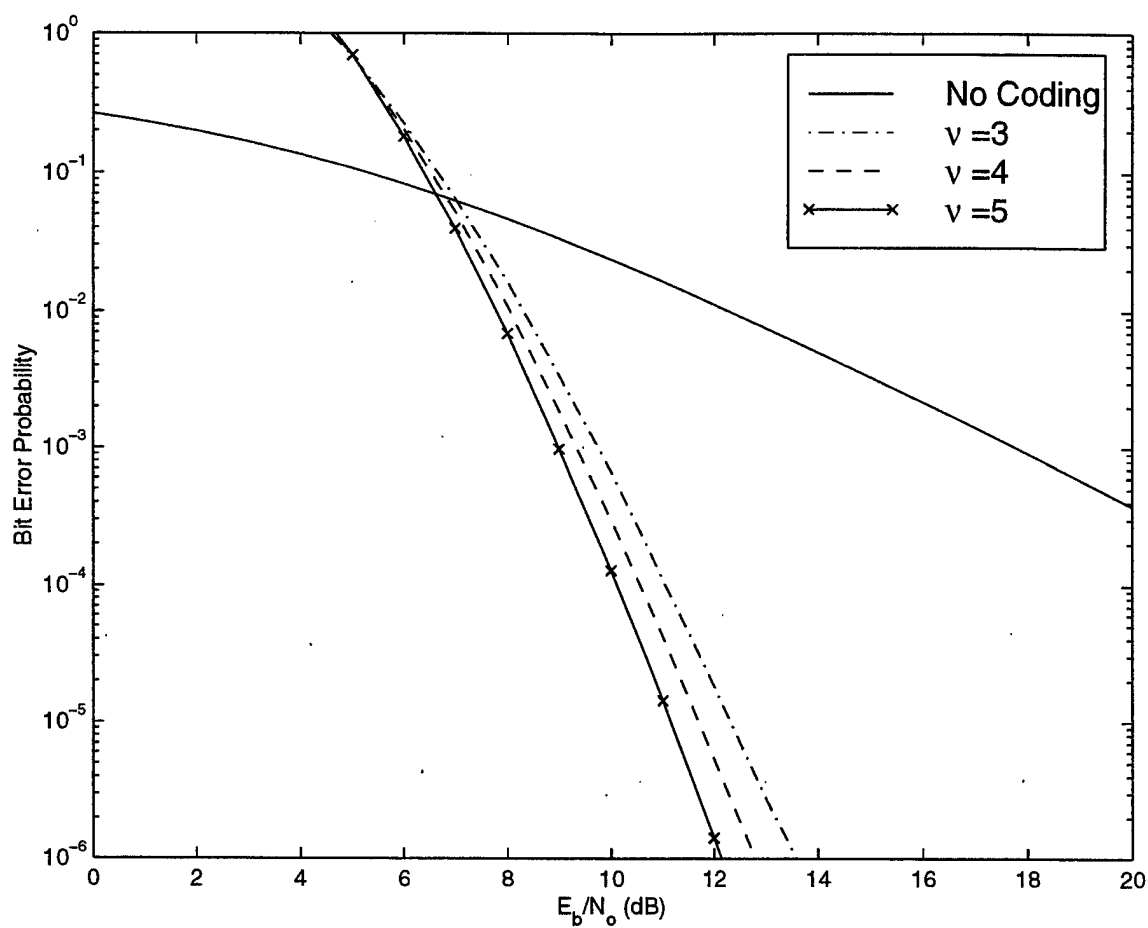


Figure 9. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 2$ and Constraint Length $v = 3, 4$ and 5.

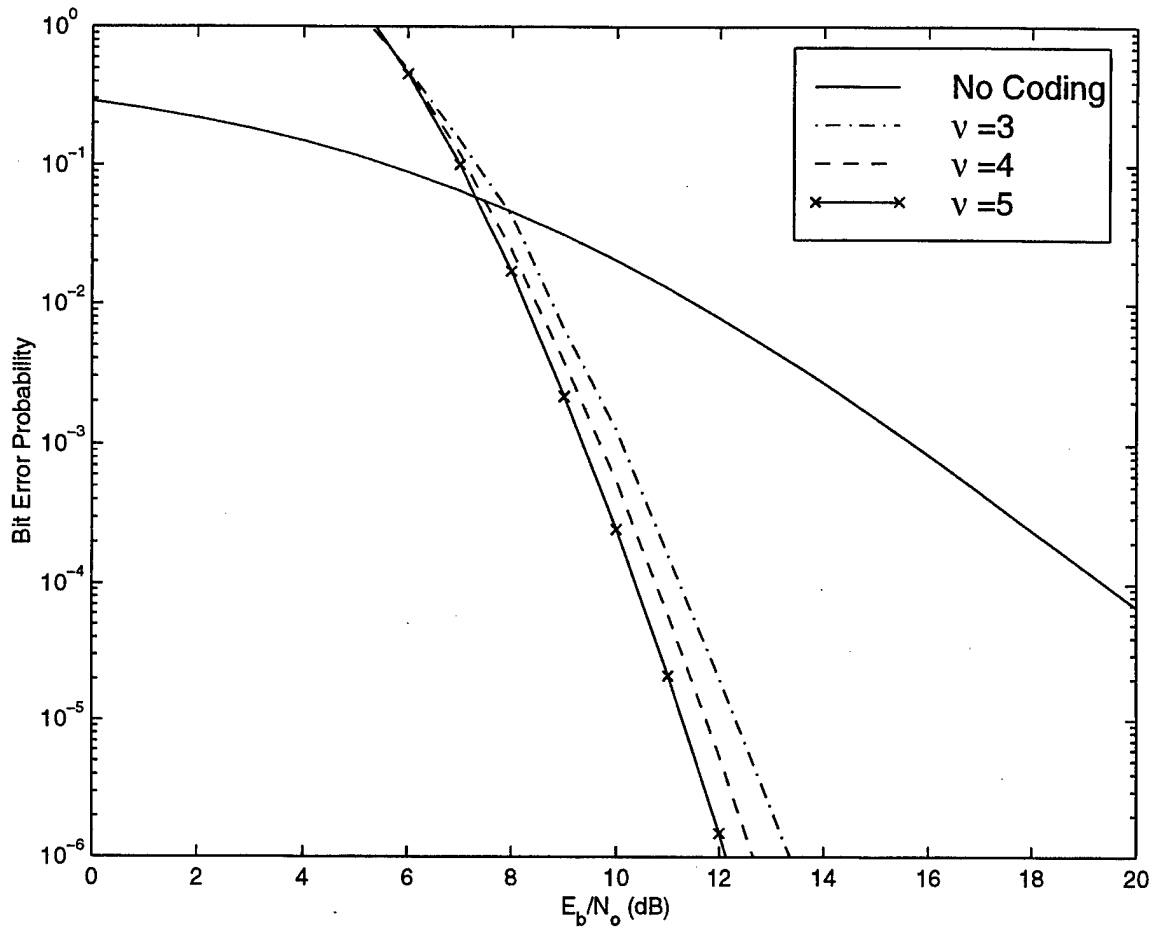


Figure 10. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 3$ and Constraint Length $v = 3, 4$ and 5 .

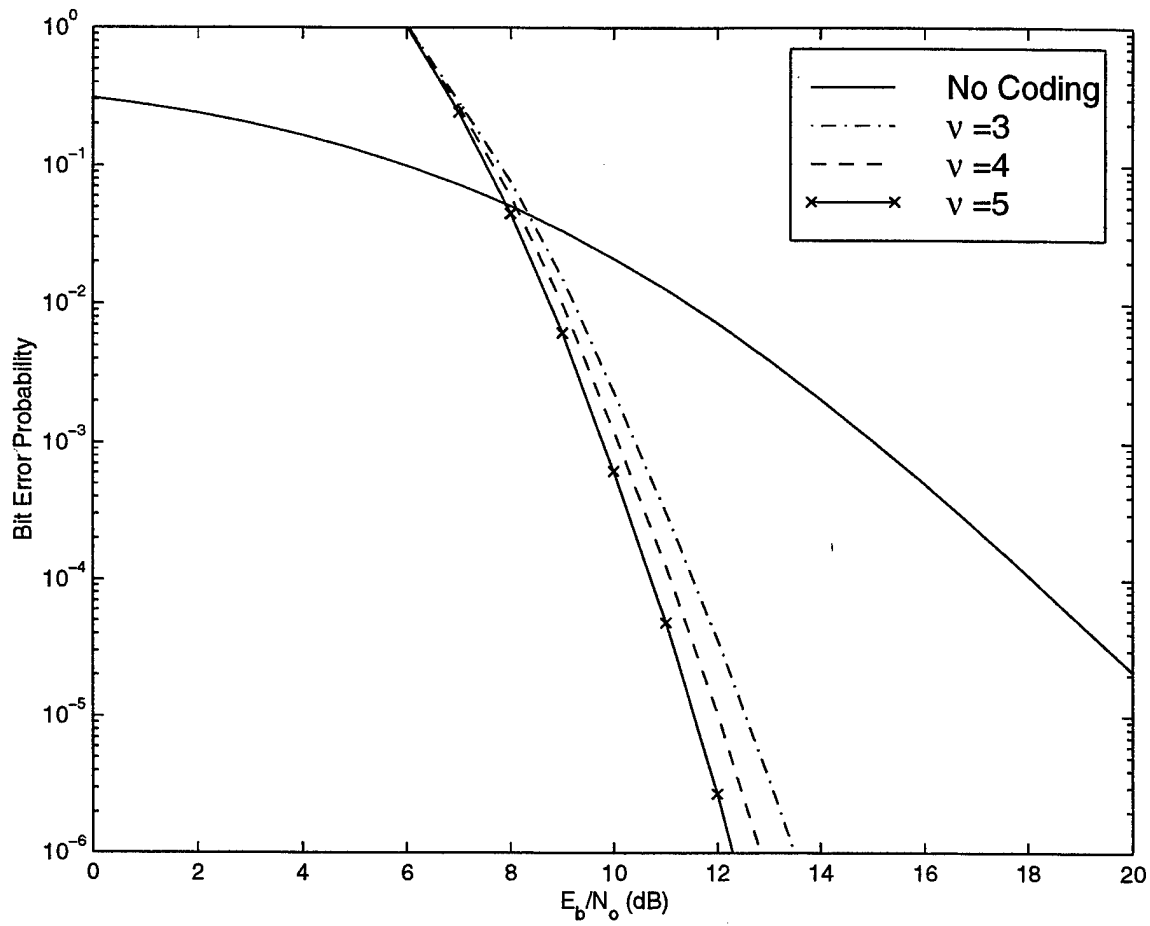


Figure 11. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 4$ and Constraint Length $v = 3, 4$ and 5.

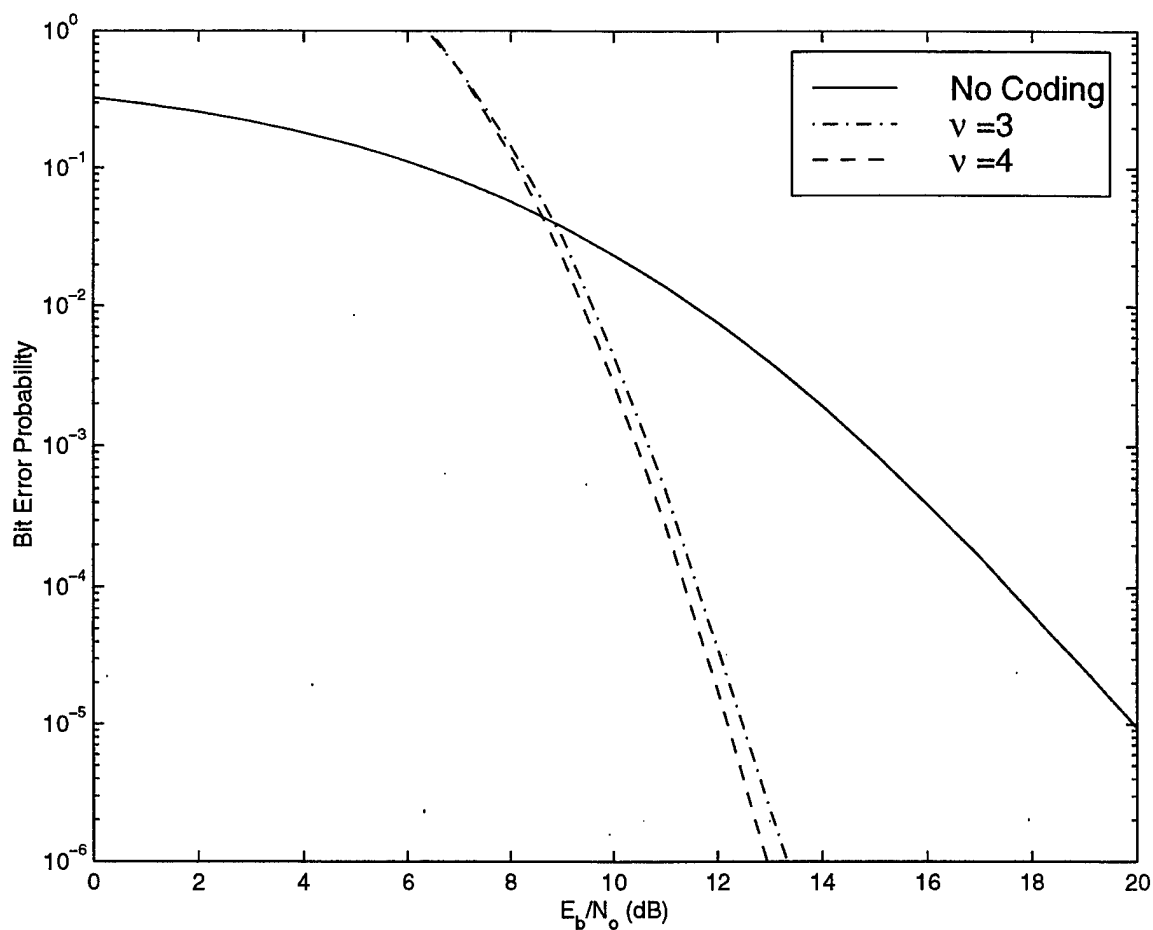


Figure 12. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 5$ and Constraint Length $v = 3$ and 4.

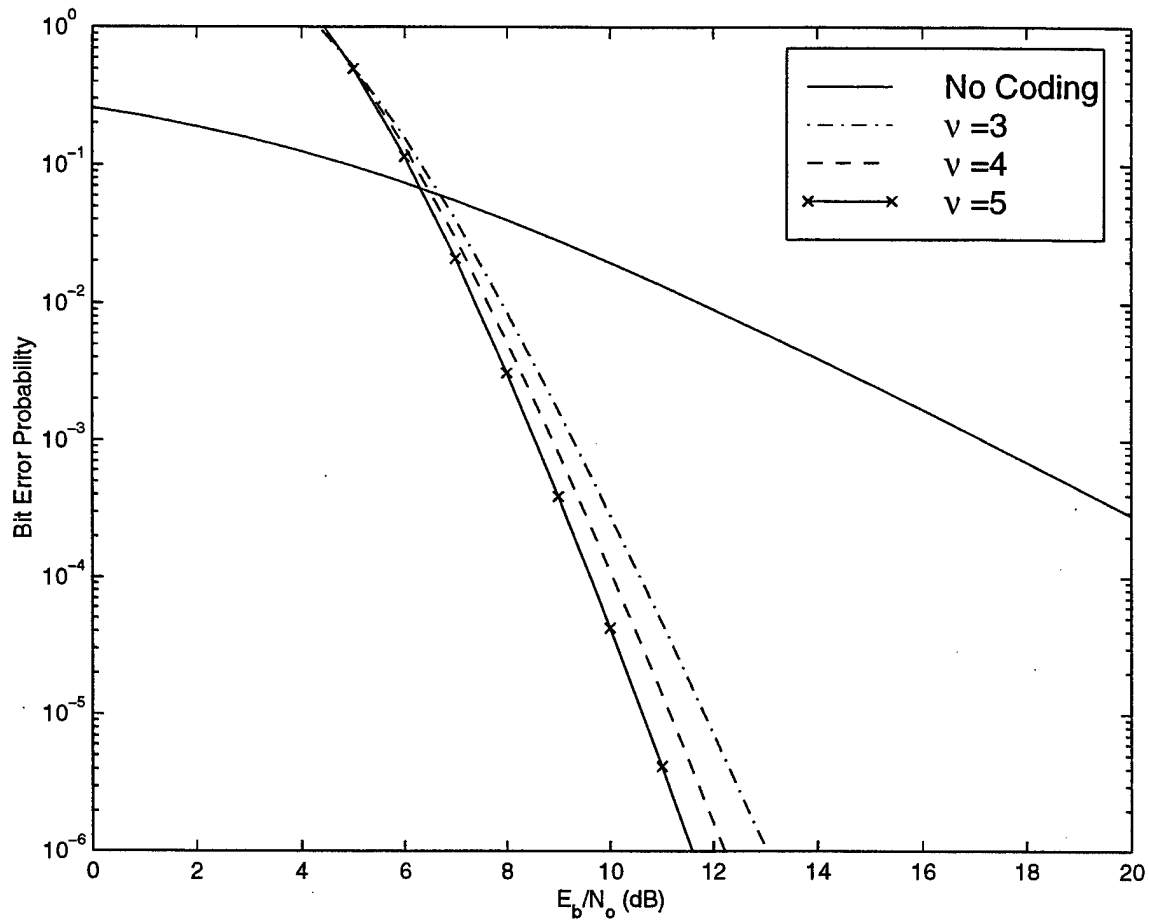


Figure 13. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 2$ and Constraint Length $v = 3, 4$ and 5.

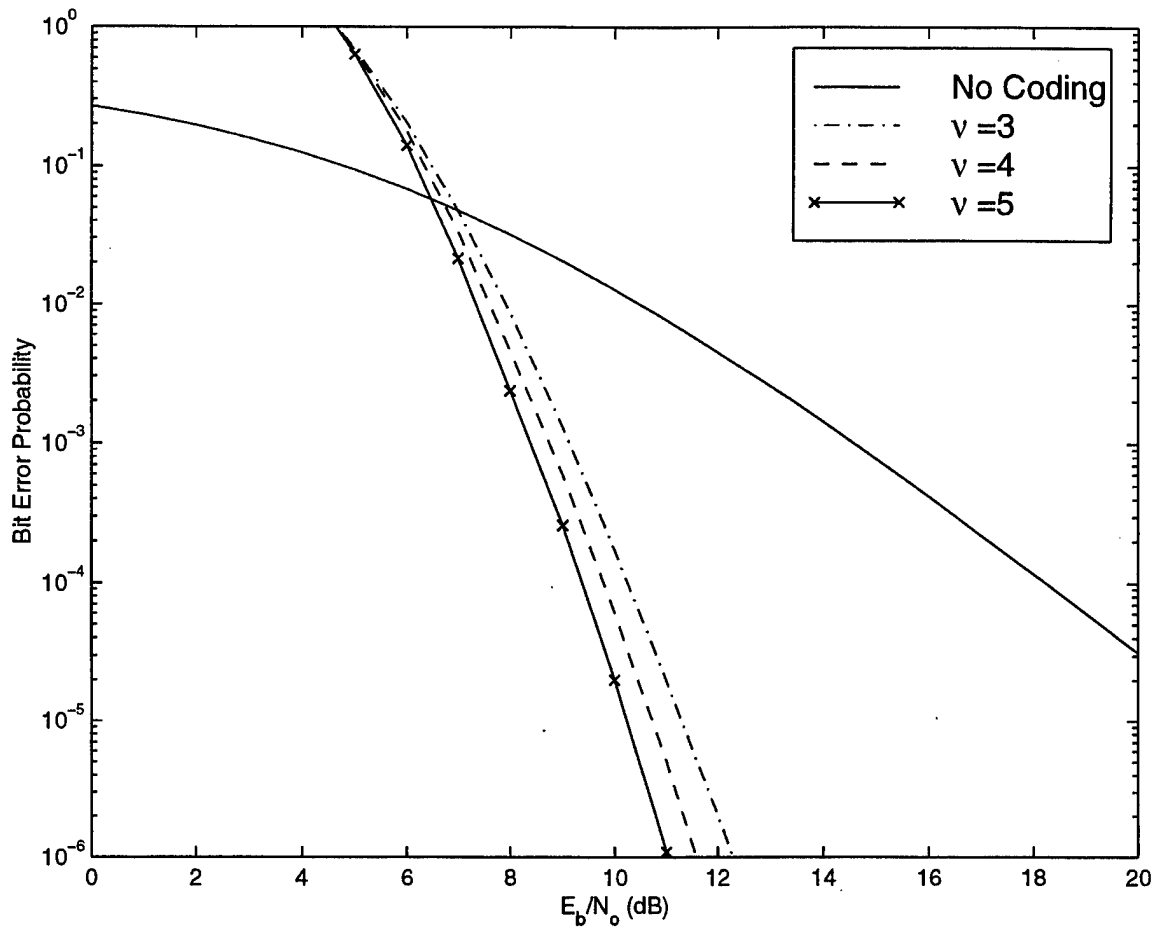


Figure 14. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 3$ and Constraint Length $v = 3, 4$ and 5.

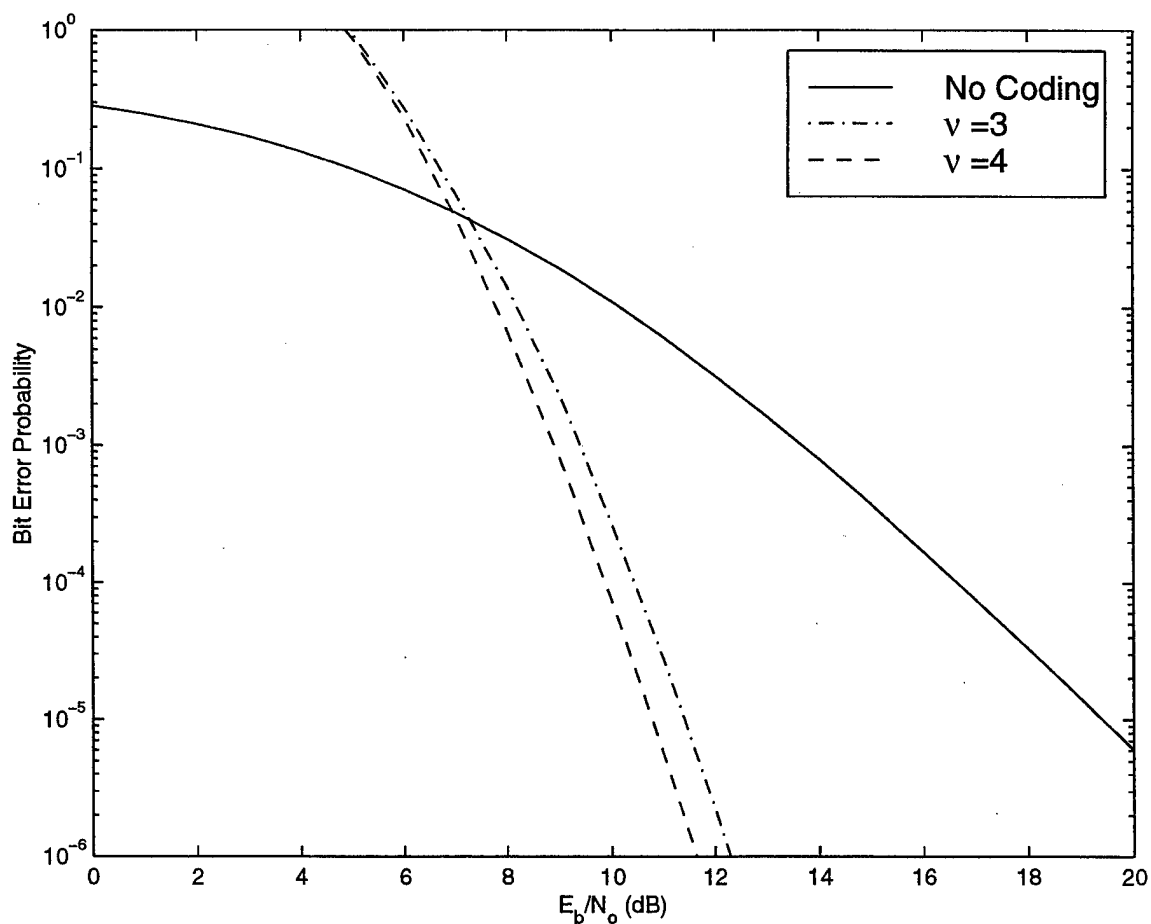


Figure 15. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 4$ and Constraint Length $v = 3$ and 4.

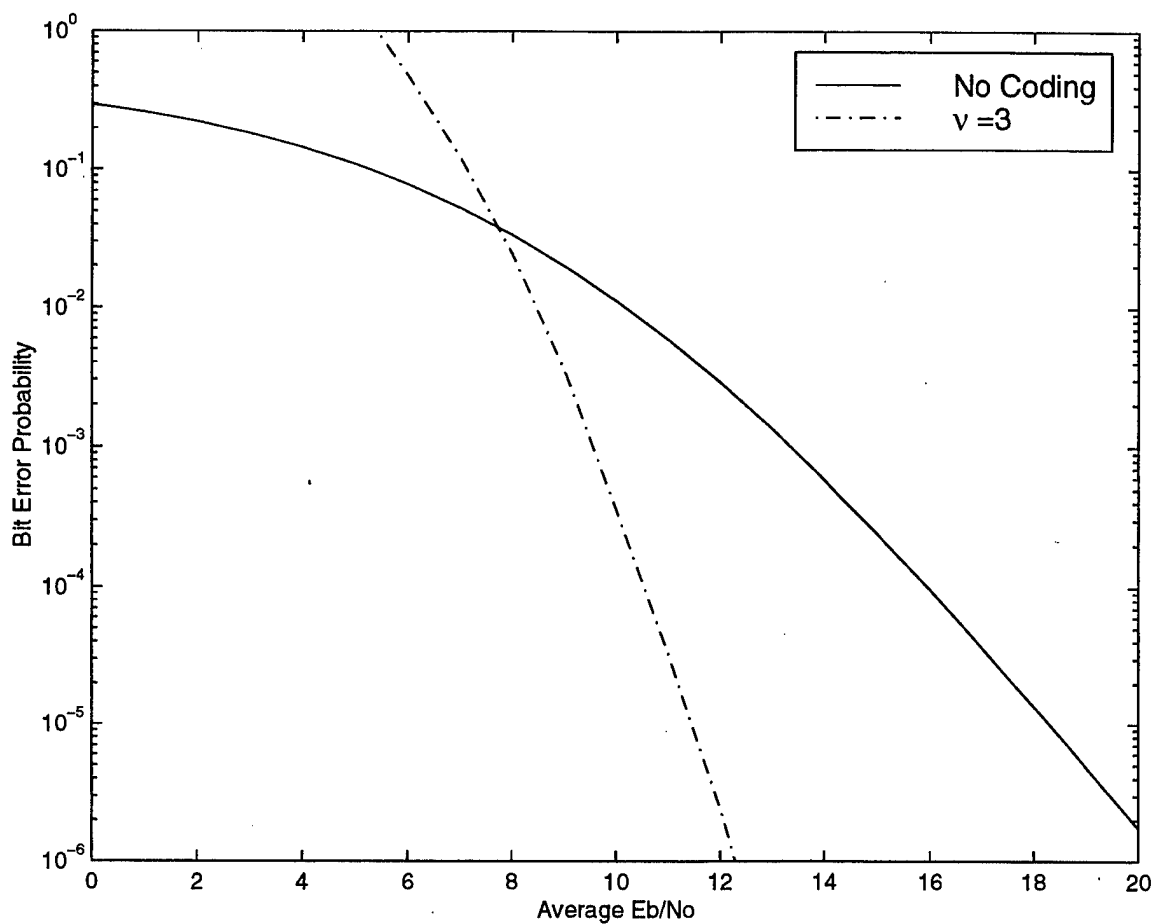


Figure 16. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 5$ and Constraint Length $v = 3$.

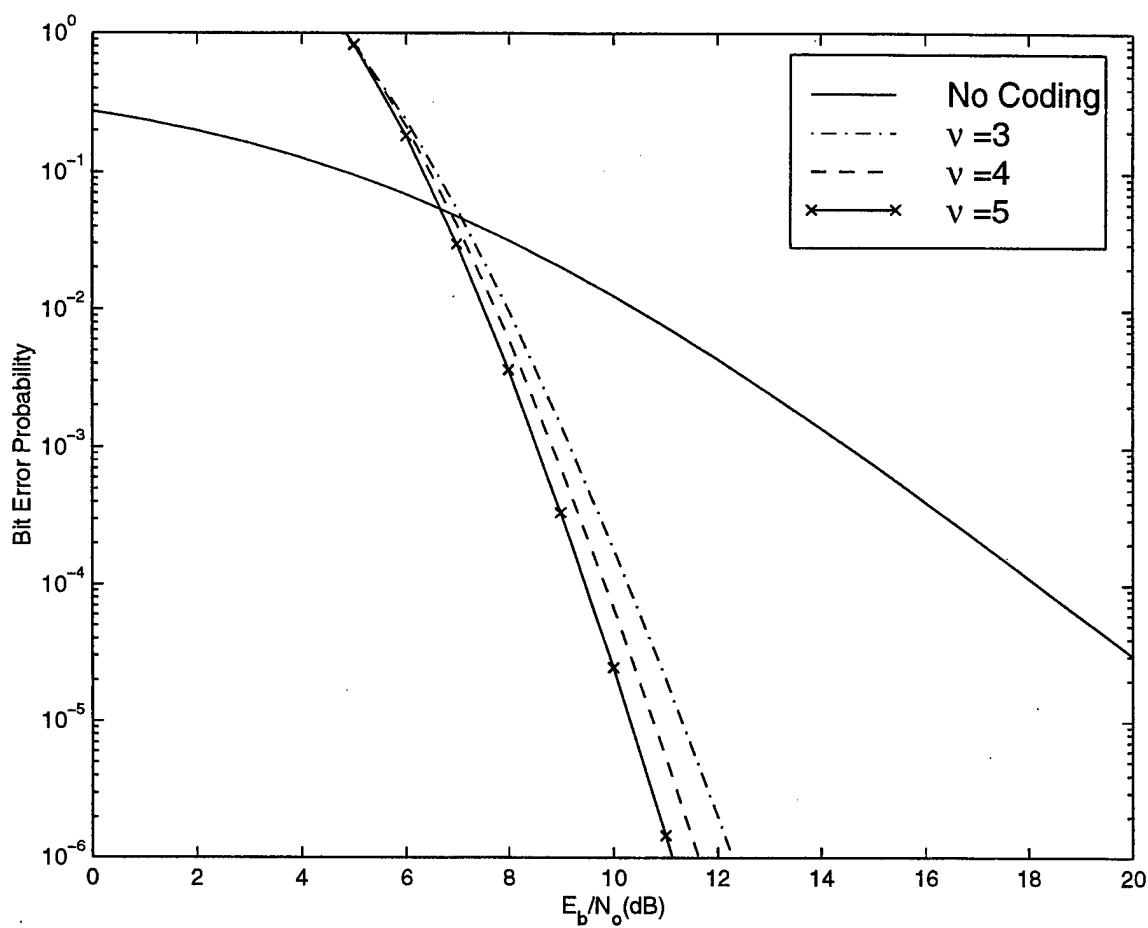


Figure 17. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Third Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 3$ and Constraint Length $v = 3, 4$ and 5.

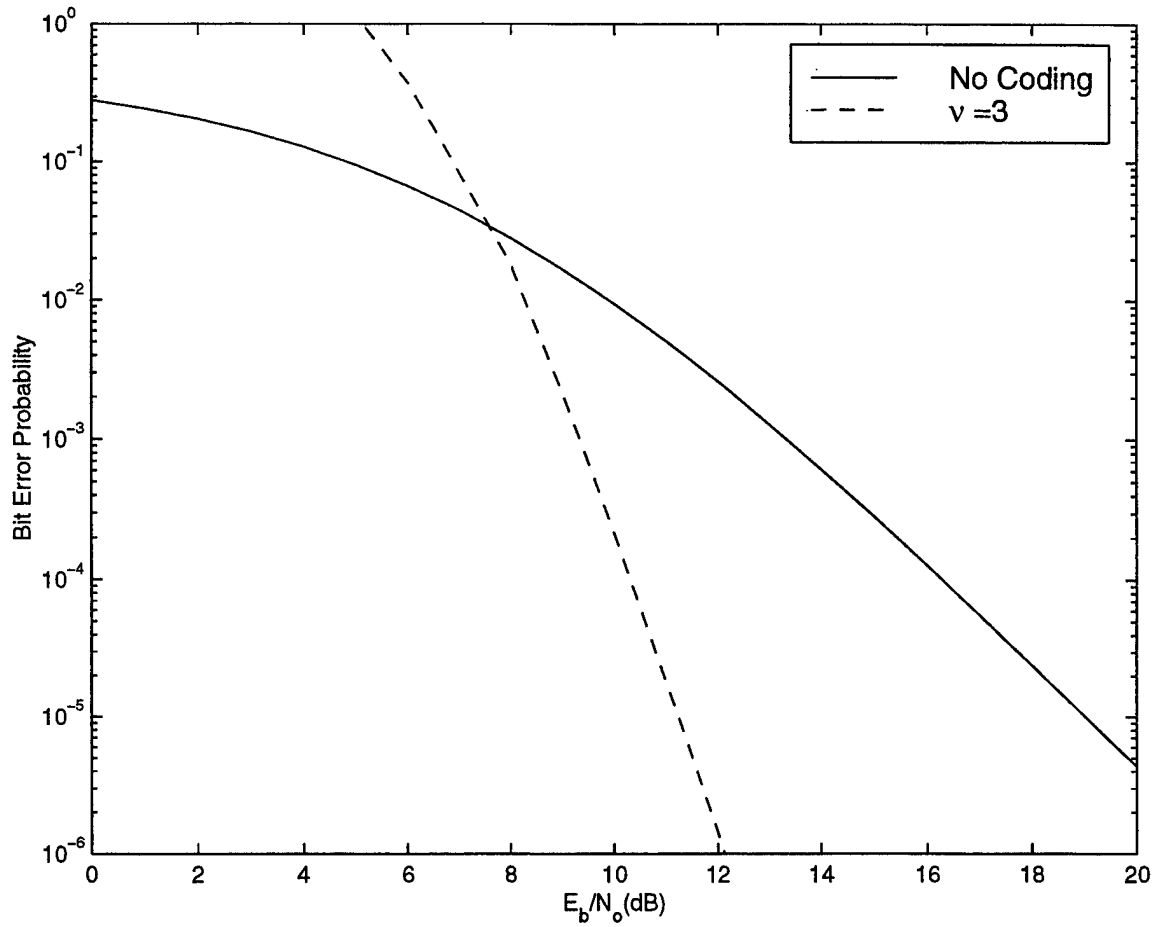


Figure 18. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Third Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversity $L = 4$ and Constraint Length $v = 3$.

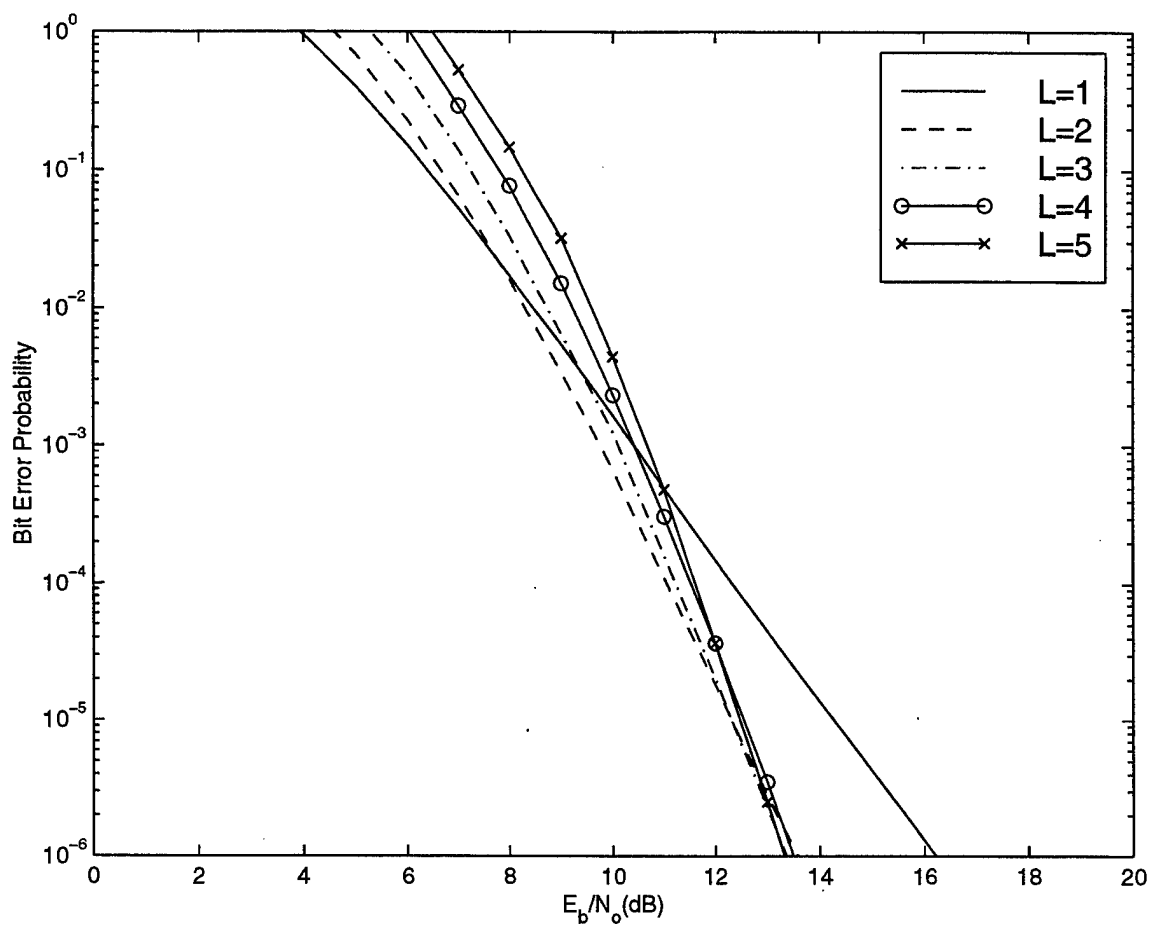


Figure 19. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 1, 2, 3, 4$ & 5 and Constraint Length $v = 3$.

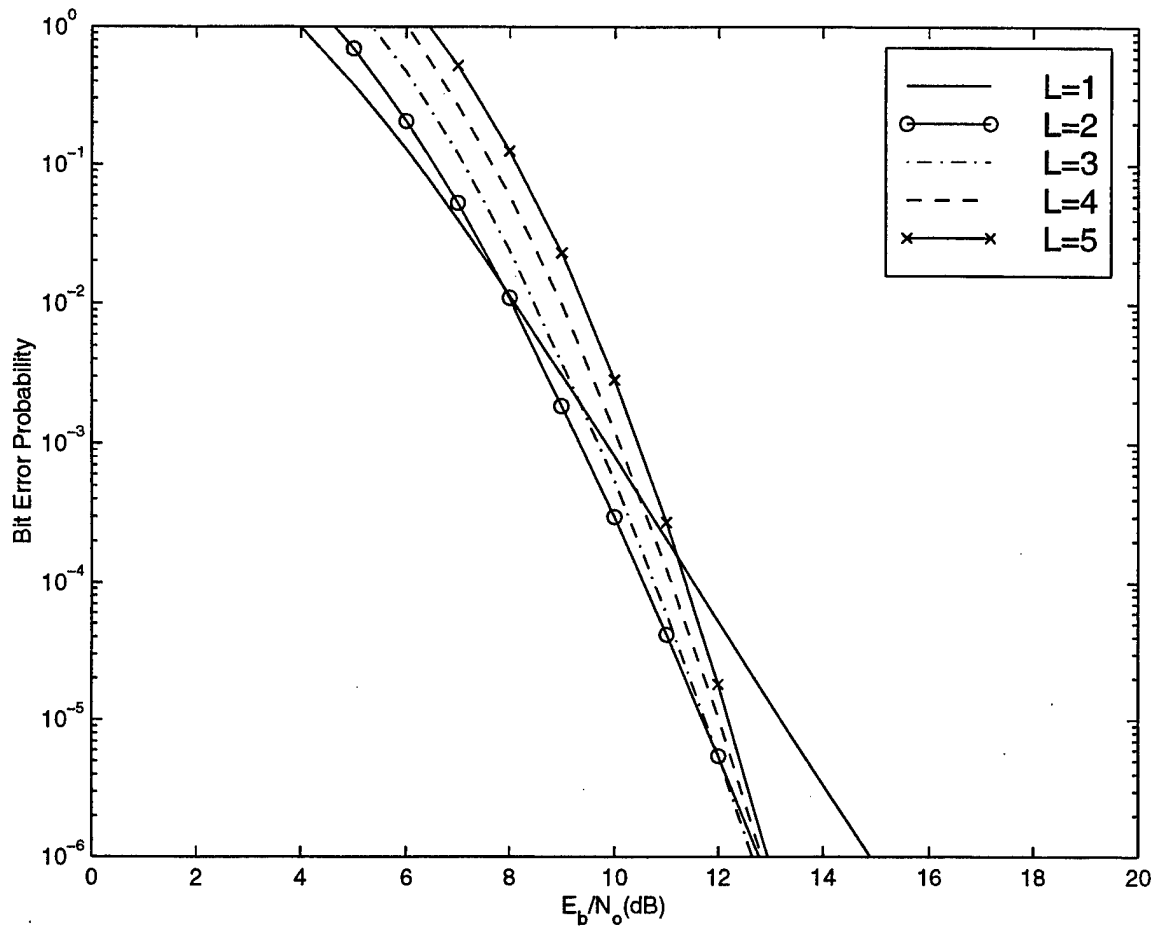


Figure 20. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 1, 2, 3, 4$ & 5 and Constraint Length $v = 4$.

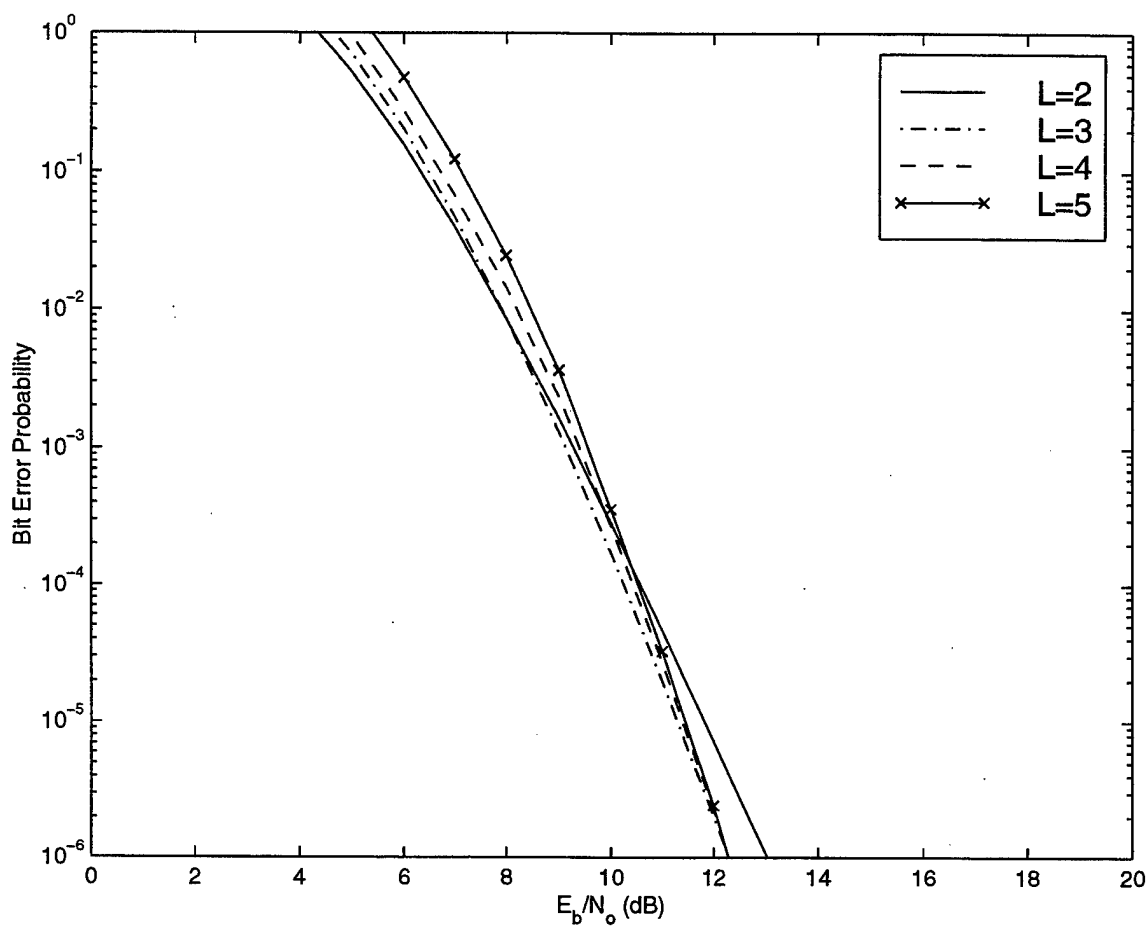


Figure 21. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 2, 3, 4$ & 5 and Constraint Length $v = 3$.

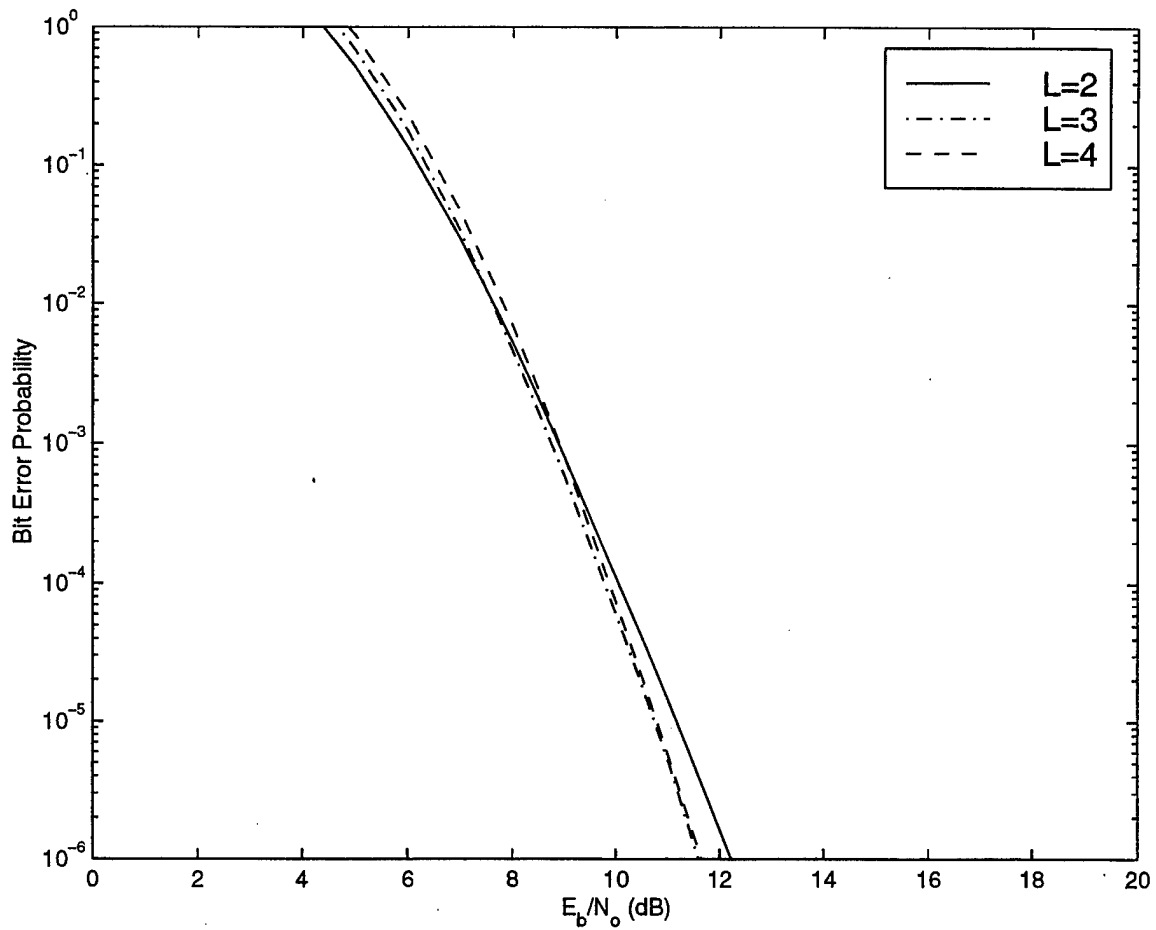


Figure 22. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Second Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 2, 3$ & 4 and Constraint Length $v = 4$.

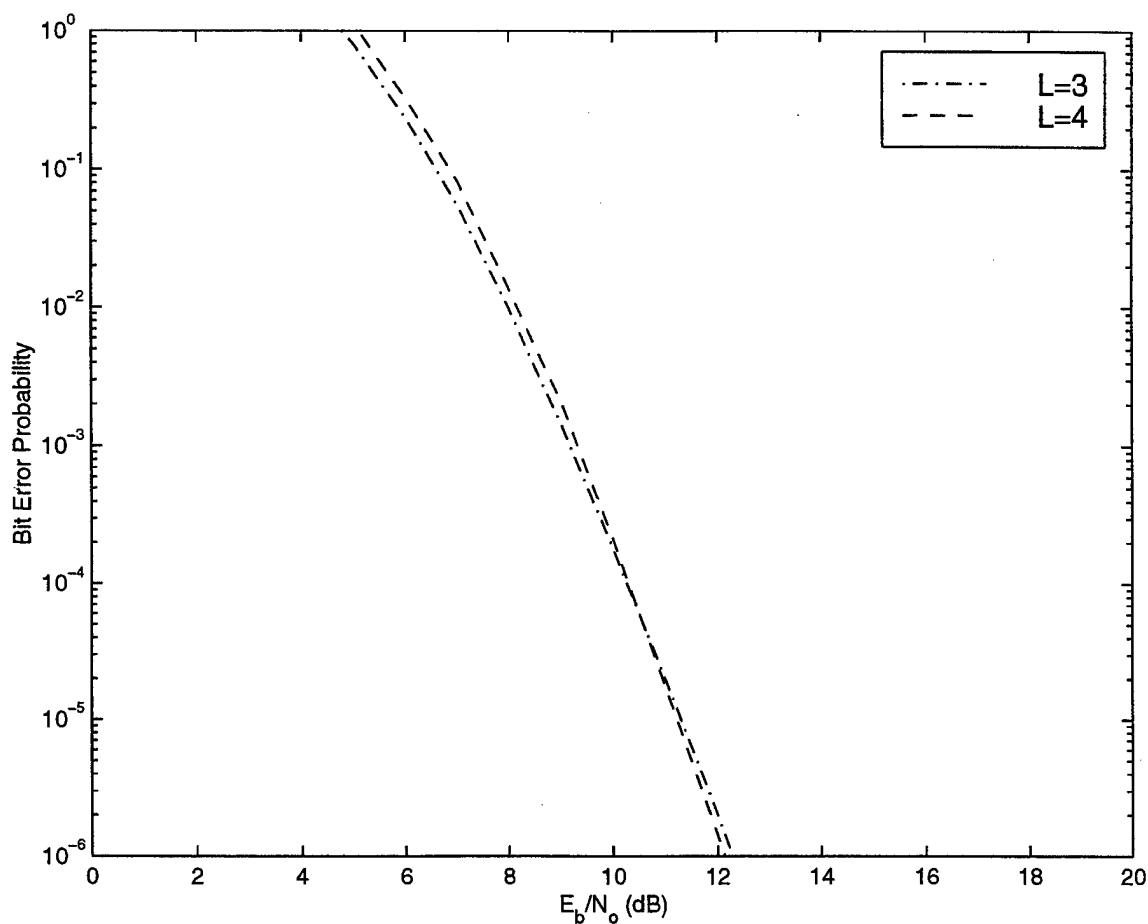


Figure 23. Performance of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Third Order Selection Combining Technique and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 3$ & 4 and Constraint Length $v=3$.

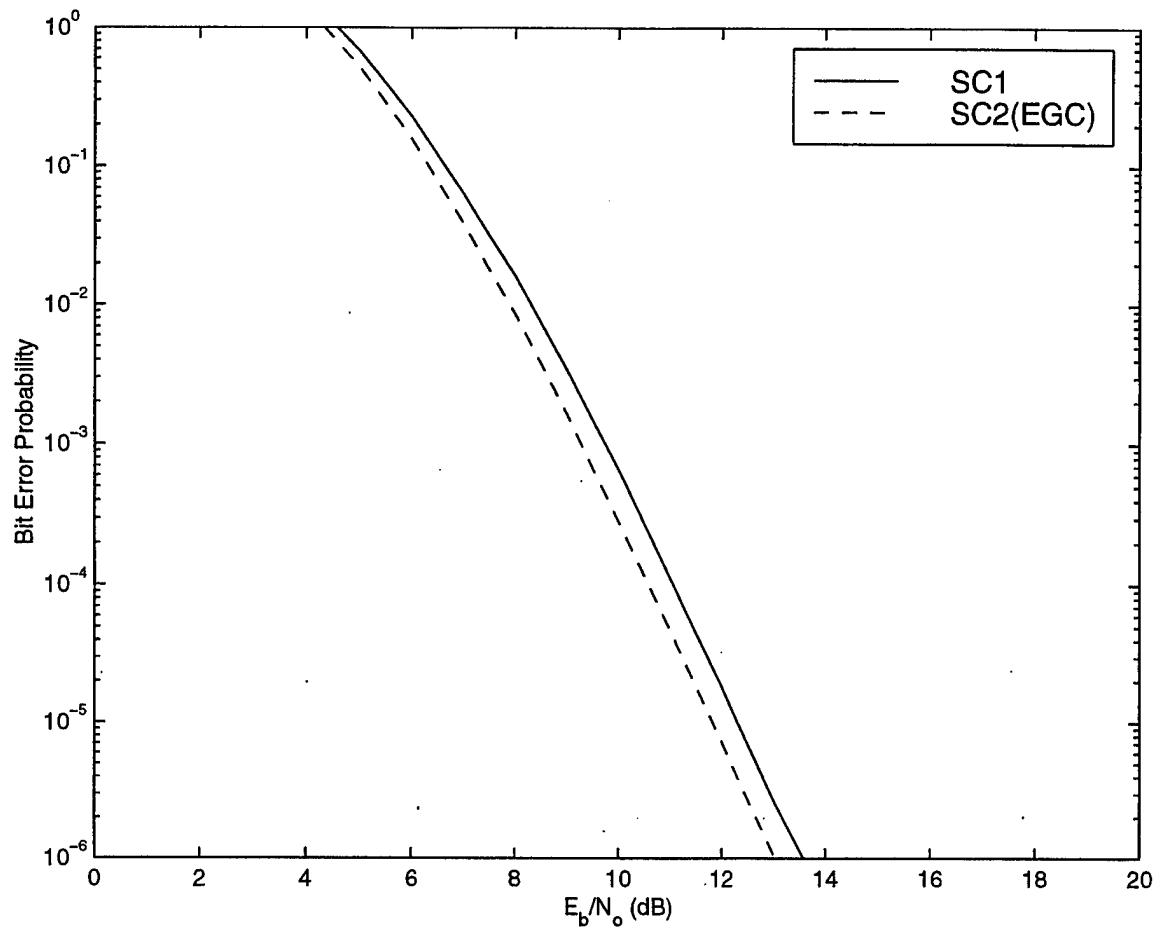


Figure 24. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 2$ and Constraint Length $v = 3$.

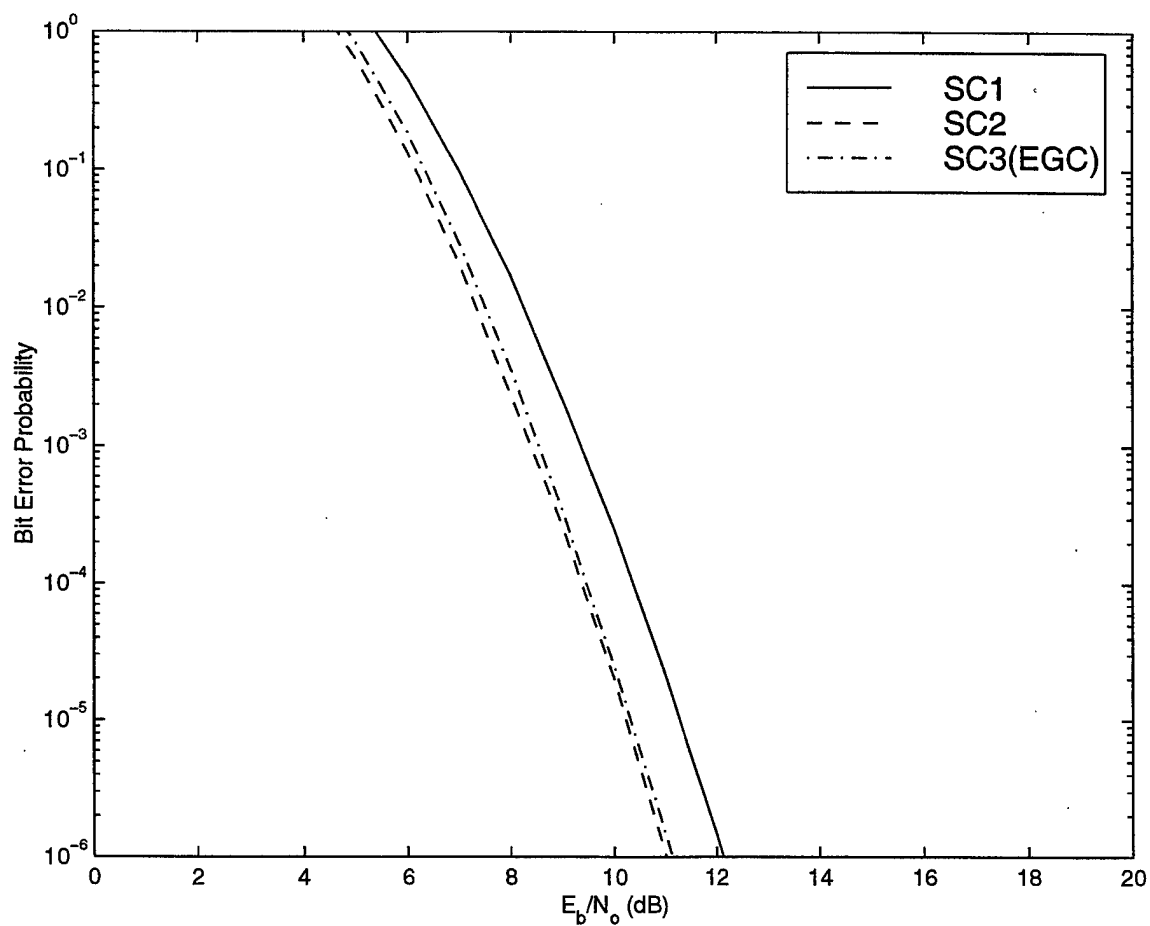


Figure 25. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 3$ and Constraint Length $v = 3$.

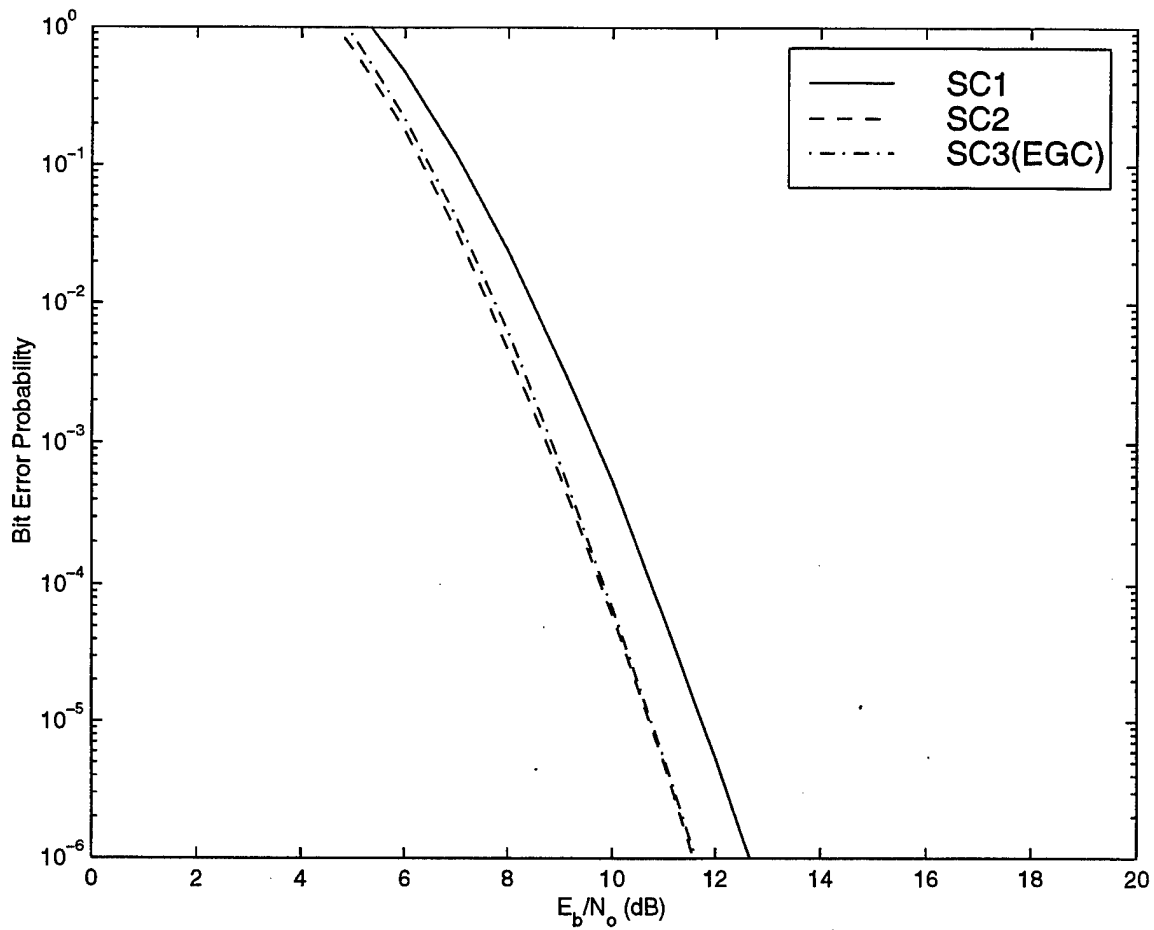


Figure 26. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 3$ and Constraint Length $v = 4$.

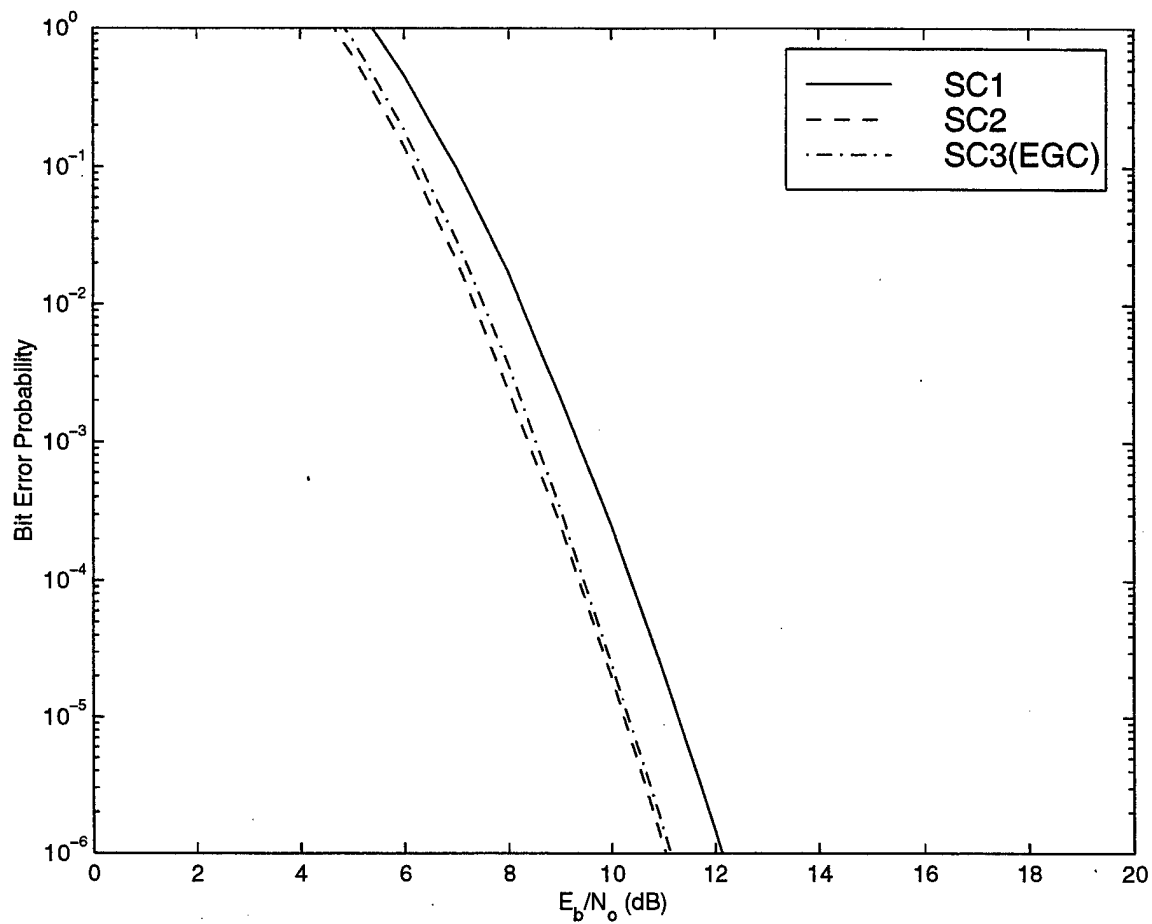


Figure 27. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading Channel With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 3$ and Constraint Length $\nu = 5$.

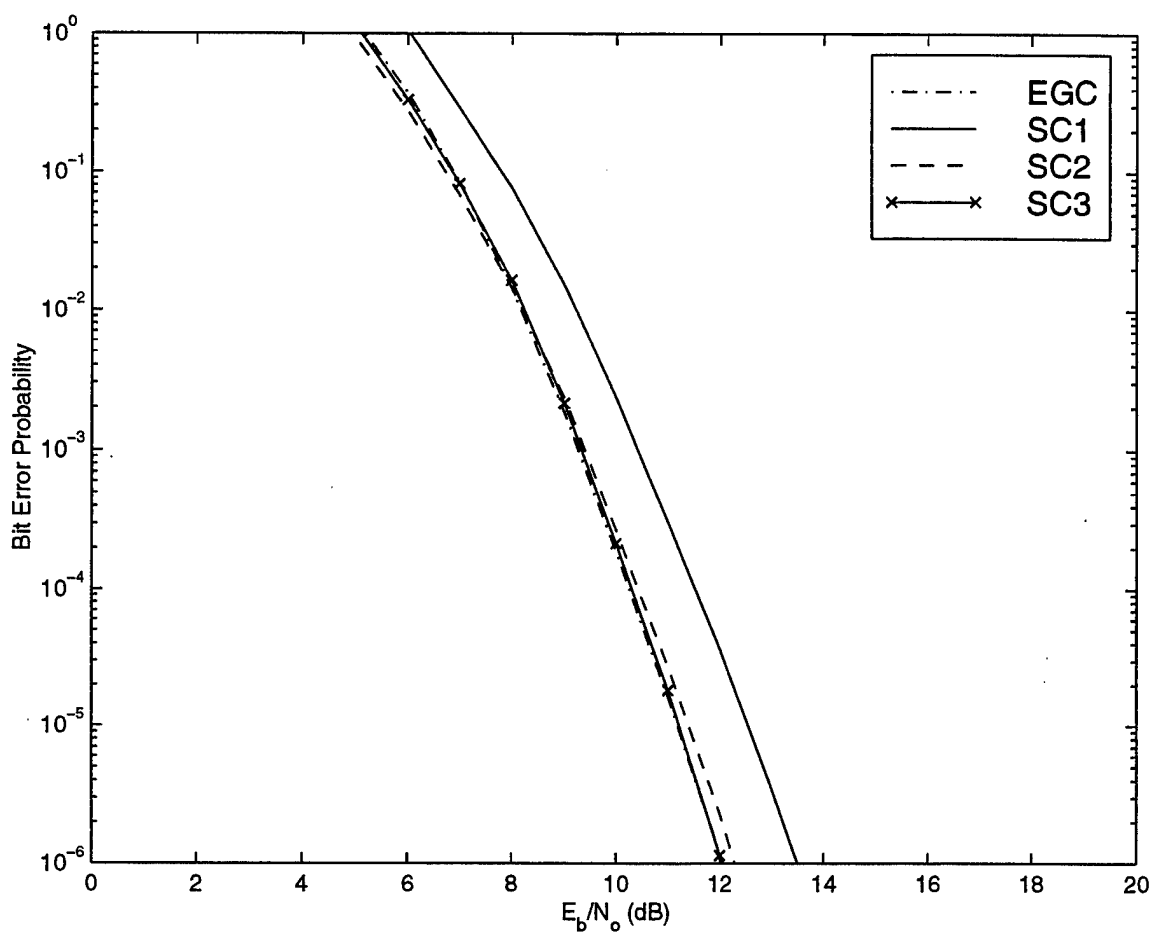


Figure 28. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 4$ and Constraint Length $v = 3$.

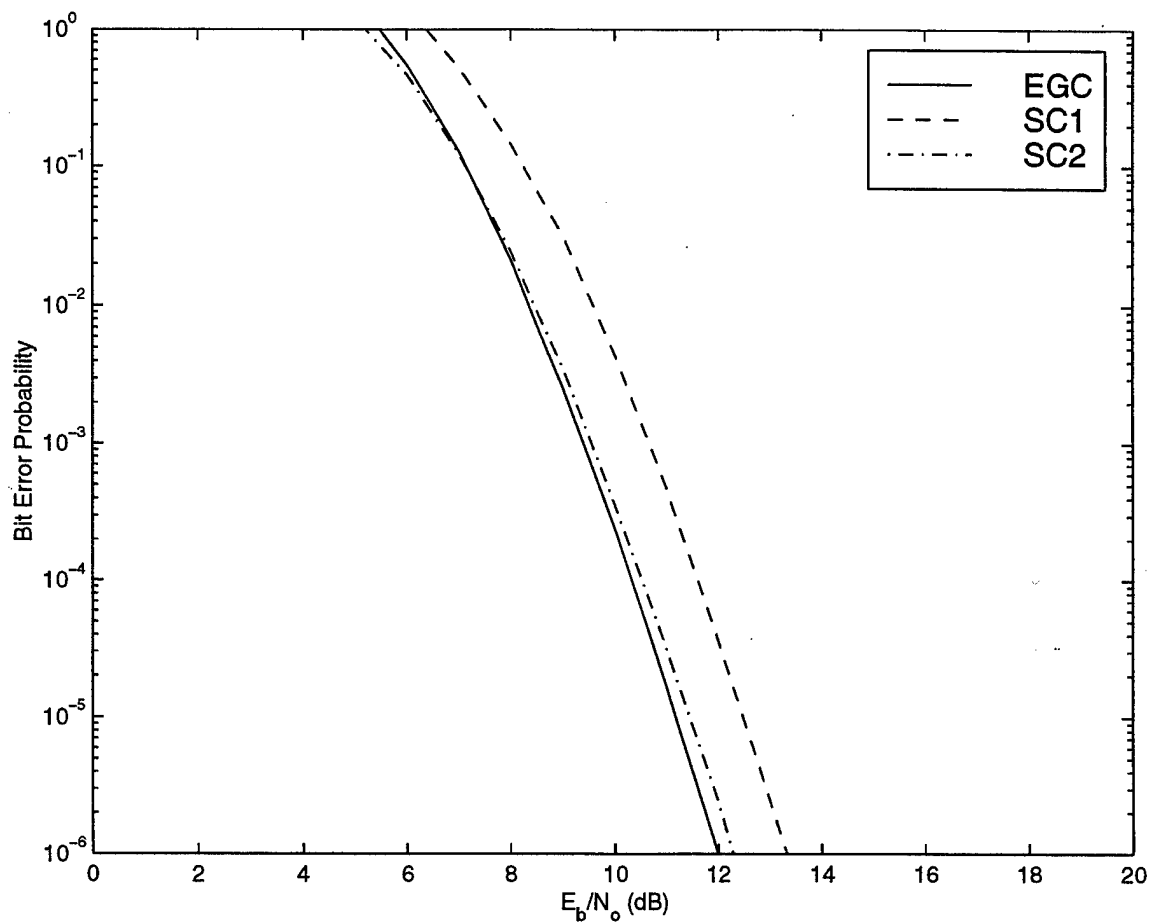


Figure 29. Comparison of Performances of a Non-Coherent Coded DPSK Receiver in Rayleigh Fading Channel With Various Combining Techniques and Soft-Decision Convolutional Decoding for Rate = 1/2, Diversities $L = 5$ and Constraint Length $\nu = 3$.

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APPENDIX A

LAPLACE TRANSFORMS OF $f(\Gamma_b)$

A. SELECTION COMBINING

L	LAPLACE TRANSFORM
1	$\frac{\alpha}{s + \alpha}$
2	$\frac{2\alpha^2}{(s + \alpha)(s + 2\alpha)}$
3	$\frac{6\alpha^3}{(s + \alpha)(s + 2\alpha)(s + 3\alpha)}$
4	$\frac{24\alpha^4}{(s + \alpha)(s + 2\alpha)(s + 3\alpha)(s + 4\alpha)}$
5	$\frac{120\alpha^5}{(s + \alpha)(s + 2\alpha)(s + 3\alpha)(s + 4\alpha)(s + 5\alpha)}$

B. SECOND ORDER SELECTION COMBINING

L	LAPLACE TRANSFORM
2	$\frac{\alpha^2}{(s + \alpha)^2}$
3	$\frac{3\alpha^3}{(s + \alpha)^2(2s + 3\alpha)}$
4	$\frac{6\alpha^4}{(s + \alpha)^2(2s + 3\alpha)(s + 2\alpha)}$
5	$\frac{30\alpha^5}{(s + \alpha)^2(2s + 3\alpha)(s + 2\alpha)(2s + 5\alpha)}$

C. THIRD ORDER SELECTION COMBINING

L	LAPLACE TRANSFORM
3	$\frac{\alpha^3}{(s + \alpha)^3}$
4	$\frac{4\alpha^4}{(s + \alpha)^3(3s + 4\alpha)}$
5	$\frac{20\alpha^5}{(s + \alpha)^3(3s + 4\alpha)(3s + 5\alpha)}$

APPENDIX B

PROCEDURE FOR OBTAINING RELIABLE DATA FOR ANALYSIS

In this appendix, we demonstrate how we obtain the reliable data for the average bit error probability for the SC technique with diversity $L=5$ and $d=10$ for the value of the average γ_b of 10 dB

A. Defining The Domain of Interest for γ_b Between 0 And 20 dB

$$q := 0.. 20$$

B. Defining the Code Rate(1/2) and Diversity (L=5)

$$r := \frac{1}{2} \quad L := 5$$

C. Defining α

$$\alpha(q) := \frac{L}{r \cdot 10^{\frac{q}{10}}}$$

D. Inputting the pdf, $f(\gamma_b)$, for L=5, SC Technique, Setting $\gamma_b=t$

$$5 \cdot \alpha(q) \cdot e^{-\alpha(q) \cdot t} \cdot (1 - e^{-\alpha(q) \cdot t})^4$$

E. Expanding The Series in (D)

$$5 \cdot \frac{\alpha(q)}{\exp(\alpha(q) \cdot t)} - 20 \cdot \frac{\alpha(q)}{\exp(\alpha(q) \cdot t)^2} + 30 \cdot \frac{\alpha(q)}{\exp(\alpha(q) \cdot t)^3} - 20 \cdot \frac{\alpha(q)}{\exp(\alpha(q) \cdot t)^4} + 5 \cdot \frac{\alpha(q)}{\exp(\alpha(q) \cdot t)^5}$$

F. Applying Symbolic Laplace Transform on Expression in (E) and Raising It to Power $d = 10$

$$\left[120 \cdot \frac{\alpha(q)^5}{[(s + \alpha(q)) \cdot [(s + 2 \cdot \alpha(q)) \cdot [(s + 3 \cdot \alpha(q)) \cdot [(s + 4 \cdot \alpha(q)) \cdot (s + 5 \cdot \alpha(q))]]]]]} \right]^{10}$$

G. Applying Symbolic Inverse Laplace Transform on Expression in (F), the pdf, $p(\Gamma_b)$, of Γ_b is equal to,

$$\frac{1953125}{72576} \cdot t^9 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^{10} - \frac{54551016845703125}{2985984} \cdot t^4 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^5$$

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$$\begin{aligned}
 & \blacksquare + \frac{63037109375}{145152} \cdot t^7 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^8 + \frac{57064084652587890625}{17915904} \cdot t^2 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^3 \\
 & \blacksquare + \frac{384899873046875}{497664} \cdot t^5 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^6 + \frac{54551016845703125}{2985984} \cdot t^4 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^5 \\
 & \blacksquare + \frac{63037109375}{145152} \cdot t^7 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^8 + \frac{384899873046875}{497664} \cdot t^5 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^6 \\
 & \blacksquare - \frac{5468750000}{243} \cdot t^6 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^7 - \frac{675200000000000}{243} \cdot t^6 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^7 \\
 & \blacksquare + \frac{11390625000}{7} \cdot t^9 \cdot \exp(-3 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^{10} + \frac{14312560000000000}{243} \cdot t^5 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^6 \\
 & \blacksquare - \frac{78202179947528076171875}{1289945088} \cdot \alpha(q) \cdot \exp(-\alpha(q) \cdot t) + \frac{5311271396728515625}{17915904} \cdot t^3 \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^4 \\
 & \blacksquare - \frac{400000000000}{189} \cdot t^8 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^9 + \frac{21953954000000000000}{2187} \cdot t^3 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^4 \\
 & \blacksquare - \frac{1671479480000000000000}{2187} \cdot t^2 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^3 + \frac{4417850722776025390625}{214990848} \cdot t \cdot \exp(-5 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^2 \\
 & \blacksquare - \frac{57064084652587890625}{17915904} \cdot t^2 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^3 + \frac{10251562500000}{7} \cdot t^7 \cdot \exp(-3 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^8 \\
 & \blacksquare + \frac{536000000000000}{567} \cdot t^7 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^8 + \frac{5311271396728515625}{17915904} \cdot t^3 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^4 \\
 & \blacksquare + \frac{1671479480000000000000}{2187} \cdot t^2 \cdot \exp(-4 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^3 + 880556679492187500 \cdot t \cdot \exp(-3 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^2 \\
 & \blacksquare + \frac{2368769933200000000000}{6561} \cdot t \cdot \exp(-4 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^2 + \frac{2368769933200000000000}{6561} \cdot t \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^2 \\
 & \blacksquare + \frac{4417850722776025390625}{214990848} \cdot t \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^2 + \frac{15770326286000000000000}{19683} \cdot \alpha(q) \cdot \exp(-4 \cdot \alpha(q) \cdot t) \\
 & \blacksquare - \frac{244140625}{48384} \cdot t^8 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^9 + \frac{6602420000000000000}{729} \cdot t^4 \cdot \exp(-4 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^5 \\
 & \blacksquare + \frac{143125600000000000}{243} \cdot t^5 \cdot \exp(-4 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^6 + \frac{1953125}{72576} \cdot t^9 \cdot \exp(-\alpha(q) \cdot t) \cdot \alpha(q)^{10} \\
 & \blacksquare + 410575078125000 \cdot t^5 \cdot \exp(-3 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^6 + \frac{78202179947528076171875}{1289945088} \cdot \alpha(q) \cdot \exp(-5 \cdot \alpha(q) \cdot t) \\
 & \blacksquare + \frac{160000000000}{567} \cdot t^9 \cdot \exp(-2 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^{10} + \frac{675200000000000}{243} \cdot t^6 \cdot \exp(-4 \cdot \alpha(q) \cdot t) \cdot \alpha(q)^7
 \end{aligned}$$

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$$\begin{aligned}
 & - \frac{15770326286000000000000}{19683} \cdot a(q) \cdot \exp(-2 \cdot a(q) \cdot t) + 38904679687500000 \cdot t^3 \cdot \exp(-3 \cdot a(q) \cdot t) \cdot a(q) \\
 & - \frac{6602420000000000000}{729} \cdot t^4 \cdot \exp(-2 \cdot a(q) \cdot t) \cdot a(q)^5 + \frac{4000000000000}{189} \cdot t^8 \cdot \exp(-4 \cdot a(q) \cdot t) \cdot a(q)^9 \\
 & + \frac{536000000000000}{567} \cdot t^7 \cdot \exp(-4 \cdot a(q) \cdot t) \cdot a(q)^8 + \frac{244140625}{48384} \cdot t^8 \cdot \exp(-5 \cdot a(q) \cdot t) \cdot a(q)^9 \\
 & + \frac{21953954000000000000}{2187} \cdot t^3 \cdot \exp(-4 \cdot a(q) \cdot t) \cdot a(q)^4 + \frac{5468750000}{243} \cdot t^6 \cdot \exp(-5 \cdot a(q) \cdot t) \cdot a(q)^7 \\
 & + \frac{16000000000}{567} \cdot t^9 \cdot \exp(-4 \cdot a(q) \cdot t) \cdot a(q)^{10}
 \end{aligned}$$

H. Determining The Dominant Domain for the pdf, $p(\Gamma_b)$, of Γ_b at the average

$\gamma_b = 10$ dB

$$pdf(q, t) := \frac{87090260000078202179947528076171875}{1289945088} \cdot a(q) \cdot \exp(-5 \cdot a(q) \cdot t) - \frac{57064084652587890625}{17915904} \cdot t^2 \cdot \exp(-a(q) \cdot t)$$

$t := 0.1, 0.2, \dots, 50$

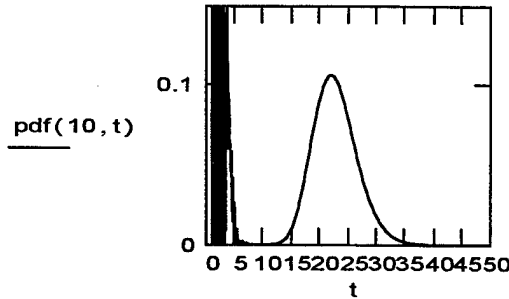


Figure B1. The Distribution of the pdf, $p(\Gamma_b)$, of Γ_b at the average $\gamma_b = 10$ dB.

From Fig. B1, the dominant domain of the pdf, $p(\Gamma_b)$, of Γ_b at the average $\gamma_b = 10$ dB is between 10 and 50. It is observed that the expression in (G), due to round-off errors, produces erroneous result for t less than 5. Hence, we select the lower and upper limits for the numerical integration to be 10 and 50, respectively.

I. Verifying That Intergration of The pdf, $p(\Gamma_b)$, of Γ_b at The Average $\gamma_b = 10$ dB Between The Lower And Upper Limits Determined in (H) Satisfies The Basic Property of pdf.

$$\int_{10}^{50} pdf(10, t) dt = 1$$

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J. Determining The Bit Error Probability at The Average $\gamma_b = 10$ dB

$$P_2(q) := \int_{10}^{50} \left[\left(\frac{1}{2} \right)^{2 \cdot 10 - 1} \cdot e^{-t} \cdot \left[\sum_{k=0}^{10-1} \frac{1}{k!} \cdot (t)^k \cdot \sum_{n=0}^{10-1-k} \frac{2 \cdot 10 - 1!}{2 \cdot 10 - n - 1! \cdot n!} \right] \right] \cdot \text{pdf}(q, t) dt$$

$$P_2(10) = 4.3077 \cdot 10^{-6}$$

The bit error probability is found to be 4.3077e-6.

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